

SERGEI ZAGNY

FOUR CANONS

piano

1981–2022

Z
N G Y
A

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Score 001

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Thanks to Keith Hammond for corrections in the English text.

Сергей Загний

Четыре канона

фортепиано

1981–2022

Z
N G Y
A

To
Victor Pavlovitch Frayonov

Виктору Павловичу Фраёнову

PREFACE

In “Four Canons” the new methods of transforming a *proposta* into a *risposta* are used. They can be described within the framework of the extended theory of canonic transformations. The author of these lines started developing this theory in connection with the present work.

BASIC CONCEPTS

Let *proposta* and *risposta* be *melodies*, i. e. monophonic sequence of notes. Canonic transformations imply a rule that defines the way of converting the *proposta* into the *risposta*. The following transformations are classical: (0) *exact repetition* (possibly with *transposition*); (1) *augmentation-diminution*; (2) *inversion*; (3) *retrograde motion*; and also the following combinations: (1)+(2), (1)+(3), (2)+(3) и (1)+(2)+(3).

Augmentation-diminution affects note durations; transposition or inversion affects pitches; retrograde motion affects the order of notes. Thus, a *note* can be defined as (d, p, n) , where d is a *duration*, p is a *pitch*, and n is a *sequence number* of a note in a melody. A *rest*, in contrast to a *sounding note*, is defined as a note with a *vacant pitch* (\emptyset), i. e. a rest is (d, \emptyset, n) . If there are several successive rests in a music score, they are considered as a single rest, the duration of which is equal to their sum.

The following is confined with the cases when each note of the *risposta* is associated with exactly one note of the *proposta*. We express a note of the *proposta* as (d, p, n) , an associated note of the *risposta* as (d', p', n') , a relation between them as $(d', p', n') = f(d, p, n)$, where f is a *rule of transformation*. When we consider only durations or only pitches, the expressions $d' = f(d)$ or $p' = f(p)$ are used. Hereafter it is meant that the rule $n' = f(n)$ which defines the order of notes is always $n' = n$, i. e. a succession of notes always remains unchanged. It is also meant that for any $p' = f(p)$ transformations, a sounding pitch converts into a sounding pitch, and a rest into a rest.

If a canon has two *rispostas*, a duration and a pitch of the *risposta-1* or of the *risposta-2* are denoted as ${}^1d'$ and ${}^1p'$, or ${}^2d'$ and ${}^2p'$ respectively.

Also we need a concept of a *melodic interval* (i). It is defined as an interval between given pitch (p) and a pitch of the *previous sounding note* (sp_{prev}), i. e. $i = p - sp_{\text{prev}}$. Melodic interval for *risposta* is denoted and defined as $i' = p' - sp'_{\text{prev}}$; for *risposta-1* as ${}^1i' = {}^1p' - {}^1sp'_{\text{prev}}$, etc.

Let us assign the following numerical values to durations: let $\text{♩} = 1$, ♩^3 (triplet eighth) = $\frac{4}{3}$, $\text{♪} = 2$, ♪^3 = $\frac{8}{3}$, $\text{♩} = 3$, $\text{♩} = 4$, etc. Durations may be matched only to positive numbers.

Numerical values for pitches we assign in the following way. The pitches are the members of chromatic scale. Neighbouring pitches differs by one. “C” of the first octave is a datum (0). Pitches above the datum associated with positive numbers, pitches below associated with negative numbers. Pitches may be matched only to integers.

CLASSICAL TRANSFORMATIONS

Classical transformations of durations are defined by the following formulas:

$d' = d$ is an exact repetition of durations;

$d' = 2d$ is double augmentation;

$d' = 3d/2$ is one-and-half augmentation;

$d' = d/3$ is triple diminution;

etc.

Classical transformations of pitches are defined by the following formulas:

$p' = p$ is the exact repetition of pitches;

$p' = p + a$ (a is an integer) is transposed repetition (a is an interval of transposition);
 $p' = -p + a$ (a is an integer) is inversion ($a/2$ is the center of inversion).

TRANSFORMATIONS USED IN "FOUR CANONS"

The transformations of durations are nonclassical in all four canons. The transformations of pitches are nonclassical in the third canon.

Canon 1. Proposta is in upper voice. The durations of the risposta are derived according to the formula $d' = 8/d$ (Fig. 1).

In respect to pitches, the risposta is an inversion of the proposta: $p' = -p + 5$ (Fig. 2).

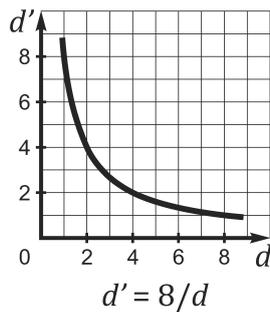


Figure 1

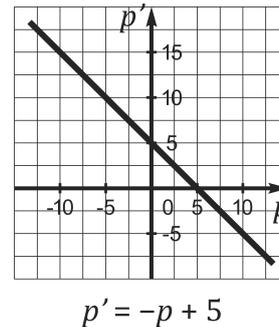


Figure 2

Canon 2. Proposta is in upper voice. The transformation of durations is as follows. Sounding notes placed directly before the rests are transformed according to the rule $d' = 1$ (all they become sixteenthths). Remaining sounding notes are transformed according to the rule $d' = 2$ (Fig. 3, all these notes become eighthths). The rests within the phrases become eighthths or dotted eighthths (measures 2–4, lower system, p. 4). The rests between the phrases are considerably longer; the durations of these rests are not predefined. The boundaries between the phrases were defined freely.

In respect to pitches, this is a canon at the lower twelfth: $p' = p - 19$ (Fig. 4).

Canon proper is only the first half of the piece. The second half is a free addition to the first.

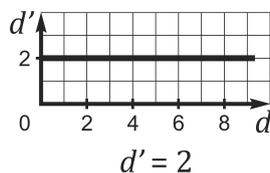


Figure 3

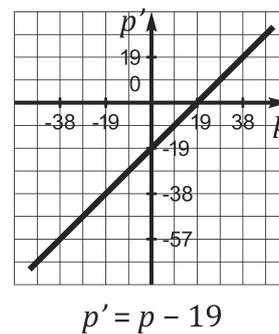


Figure 4

Canon 3. Proposta is in lower voice, two rispostas are in upper voices. Rispostas doubles each other rhythmically, so for both the rule for durations is the same. For sounding notes the formula is $d' = d + 3$ (Fig. 5, to each note dotted eighth is added). For rests the formula is $d' = d$ (rests remain unchanged).

Risposta-1 (middle voice) starts from E of the first octave. The rule for all remaining pitches of this risposta is ${}^1i' = i + \text{sgn}(i) \times 7$ (Fig. 6). (Function $\text{sgn}(i)$ is defined as follows: $\text{sgn}(i) = -1$ if $i < 0$; $\text{sgn}(i) = 1$ if $i > 0$; $\text{sgn}(i) = 0$ if $i = 0$.) In other words, the direction of melodic intervals remain intact and each interval is enlarged by perfect fifth.

The same rule may be expressed without usage of the concept of melodic interval, but the formula becomes noticeably more complicated: ${}^1p' = p - sp_{\text{prev}} + \text{sgn}(p - sp_{\text{prev}}) \times 7 + {}^1sp'_{\text{prev}}$. At the same time this expression shows explicitly that the pitch of the note of the risposta depends not only on *respective note* of proposta but on the *previous sounding notes* of proposta and risposta-1.

Risposta-2 (upper voice) starts from A of the first octave. This pitch has been chosen freely. The rule for all remaining pitches is ${}^2p' = {}^1p' + |i| - 4$. In other words, the pitch of the risposta-1 that sounds simultaneously is added to absolute value of melodic interval of proposta, then the resulting interval is decreases by major third. (Sometimes risposta-2 sounds below risposta-1.)

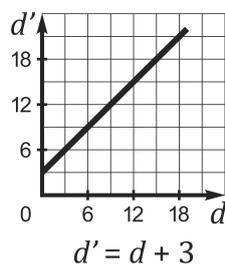


Figure 5

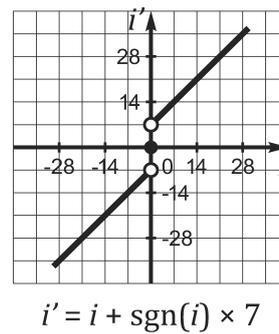


Figure 6

Canon 4. Proposta is in lower voice. The durations of the risposta are derived according to the formula $d' = |-d^2 + 8d - 6|$ (Fig. 7). The “inconvenient” values are rounded. For instance, $2\frac{8}{9}$, which is derived from the triplet eighth ($1\frac{1}{3}$) is interpreted as dotted eighth (3).

In respect to pitches, the risposta is an inversion of the proposta, as in Canon 1: $p' = -p + 5$.

Technically *Canon 4* consists of two canons, let call them canon-A and canon-B. The latter is placed “inside” the former. The piece has three sections. Canon-A is interrupted at the end of the first section and resumed in the third. Canon-B is located in the second section of the piece.

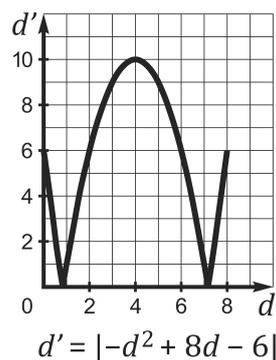


Figure 7

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ПРЕДИСЛОВИЕ

В «Четырёх канонах» применены новые способы преобразования пропосты в респосту. Их можно описать в рамках расширенной теории канонических преобразований, которую автор этих строк начал разрабатывать, задумывая это сочинение.

ОСНОВНЫЕ ПОНЯТИЯ

Пусть *пропоста* и *респоста* — это некие мелодии, т.е. одноголосные последовательности, состоящие из нот. Каноническое преобразование предполагает некоторое правило, определяющее, каким образом пропоста превращается в респосту. Классическими преобразованиями являются: (0) *точное повторение* (возможно, с *транспозицией*); (1) *увеличение-уменьшение*; (2) *обращение*; (3) *ракоход*; а также следующие комбинации: (1)+(2), (1)+(3), (2)+(3) и (1)+(2)+(3).

В случае увеличения-уменьшения меняются длительности нот; в случае транспозиции или обращения меняются высоты; в случае ракохода меняется порядок нот. Следовательно, мы можем определить ноту как (d, p, n) , где d — *длительность*, p — *высота*, и n — *порядковый номер* ноты в мелодии. Паузу, в отличие от звучащей ноты, определим как ноту с *отсутствующей высотой* (\emptyset), т.е. пауза — это (d, \emptyset, n) . Если в нотном тексте выписаны несколько пауз подряд, они рассматриваются как одна пауза, длительность которой равна их сумме.

Ограничимся случаями, когда каждой ноте респосты соответствует в точности одна нота пропосты. Ноту пропосты обозначим как (d, p, n) , соответствующую ей ноту респосты — как (d', p', n') , а отношение между ними — как $(d', p', n') = f(d, p, n)$, где f — *правило преобразования*. Когда мы рассматриваем только длительности или только высоты, мы пишем $d' = f(d)$ или $p' = f(p)$. Далее подразумевается, что правило $n' = f(n)$, определяющее порядок нот, всегда имеет вид $n' = n$, т.е. последовательность нот всегда остаётся неизменной. Также подразумевается, что при любых преобразованиях $p' = f(p)$ звучащая нота превращается в звучащую ноту, а пауза — в паузу.

Если в каноне две респосты, то для респосты-1 длительность и высоту её ноты обозначим как ${}^1d'$ и ${}^1p'$. Обозначения для респосты-2: ${}^2d'$ и ${}^2p'$.

Также нам понадобится понятие *мелодического интервала* (i). Определим его как интервал между данной высотой (p) и высотой *предшествующей звучащей ноты* (sp_{prev}), т.е. $i = p - sp_{\text{prev}}$. Мелодический интервал для респосты обозначается и определяется как $i' = p' - sp'_{\text{prev}}$; для респосты-1 — как ${}^1i' = {}^1p' - {}^1sp'_{\text{prev}}$, и т.д.

Длительностям назначим следующие числовые значения: пусть $\text{♪} = 1$, $\text{♩} = \frac{2}{3}$ (триольная восьмая) $= \frac{4}{3}$, $\text{♪} = 2$, $\text{♫} = \frac{8}{3}$, $\text{♬} = 3$, $\text{♭} = 4$, и т.д. Длительностям могут соответствовать только положительные числа.

Числовые значения для высот определим следующим образом. Звукоряд, которому принадлежат высоты — хроматическая гамма. Соседние высоты различаются на единицу. До первой октавы — точка отсчёта (0). Высоты выше этой точки — положительные числа, высоты ниже этой точки — отрицательные. Высотам могут соответствовать только целые числа.

КЛАССИЧЕСКИЕ ПРЕОБРАЗОВАНИЯ

Классические преобразования длительностей определяются следующими формулами:

$d' = d$ — точное повторение длительностей;

$d' = 2d$ — двукратное увеличение;

$d' = 3d/2$ — полуторакратное увеличение;

$d' = d/3$ — трёхкратное уменьшение;
и т. д.

Классические преобразования высот определяются следующими формулами:

$p' = p$ — точное повторение высот;

$p' = p + a$ (a — целое число) — транспонированное повторение (a — интервал транспозиции);

$p' = -p + a$ (a — целое число) — обращение ($a/2$ — центр обращения).

ПРЕОБРАЗОВАНИЯ, ИСПОЛЬЗОВАННЫЕ В «ЧЕТЫРЁХ КАНОНАХ»

Во всех четырёх канонах неклассическими являются преобразования длительностей; в третьем — также преобразования высот.

Первый канон. Пропоста — верхний голос. Длительности респосты выведены по формуле $d' = 8/d$ (рис. 1).

В отношении высот пропоста является обращением пропосты: $p' = -p + 5$ (рис. 2).

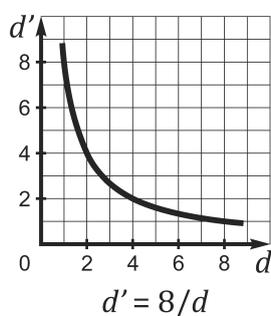


Рис. 1

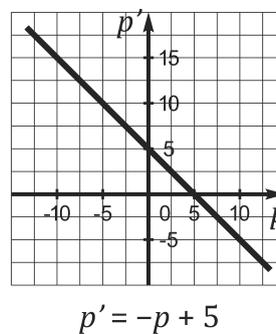


Рис. 2

Второй канон. Пропоста — верхний голос. Преобразование длительностей таково. Звучащие ноты, стоящие непосредственно перед паузами, преобразованы по правилу $d' = 1$ (все они становятся шестнадцатыми); остальные звучащие ноты преобразованы по правилу $d' = 2$ (рис. 3, все эти ноты становятся восьмыми). Паузы внутри фраз становятся восьмыми либо восьмыми с точкой (нижняя система, такты 2–4 на с. 4). Паузы между фразами существенно длиннее, их протяжённость не predetermined. Границы между фразами определялись произвольно.

В отношении высот — это канон в нижнюю дуодециму: $p' = p - 19$ (рис. 4).

Собственно каноном является только первая половина пьесы. Вторая половина — это свободное дополнение к первой.

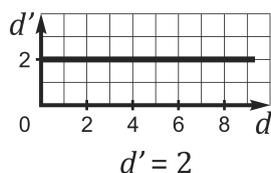


Рис. 3

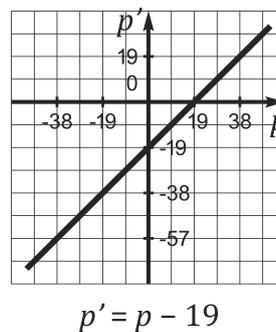


Рис. 4

Третий канон. Пропоста — нижний голос, две респосты помещены в верхних голосах. Ритмически респосты дублируют друг друга, соответственно, правила для длительностей для обеих респост одинаковы. Для звучащих нот: $d' = d + 3$ (рис. 5, к каждой ноте прибавляется восьмая с точкой); для пауз: $d' = d$ (паузы остаются неизменными).

Респоста-1 (средний голос) начинается от *ми* первой октавы. Для всех остальных высот этой респосты правило таково: ${}^1i' = i + \text{sgn}(i) \times 7$ (рис. 6). (Функция $\text{sgn}(i)$ определяется так: $\text{sgn}(i) = -1$, если $i < 0$; $\text{sgn}(i) = 1$, если $i > 0$; $\text{sgn}(i) = 0$, если $i = 0$.) Иначе говоря, направление мелодических интервалов сохраняется и каждый интервал расширяется на чистую квинту.

Если то же самое правило записать без использования понятия мелодического интервала, то формула становится заметно сложнее: ${}^1p' = p - sp_{\text{prev}} + \text{sgn}(p - sp_{\text{prev}}) \times 7 + {}^1sp'_{\text{prev}}$. В то же время такая запись делает наглядным тот факт, что высота ноты респосты зависит не только от соответствующей ноты пропосты, но также от предшествующих звучащих нот пропосты и самой респосты-1.

Респоста-2 (верхний голос) начинается от *ля* первой октавы. Эта высота выбрана произвольно. Для всех остальных высот правило таково: ${}^2p' = {}^1p' + |i| - 4$. Иначе говоря, к высоте ноты респосты-1, звучащей одновременно, прибавляется мелодический интервал пропосты, взятый в абсолютном значении, и полученная сумма уменьшается на большую терцию. (Иногда респоста-2 звучит ниже респосты-1.)

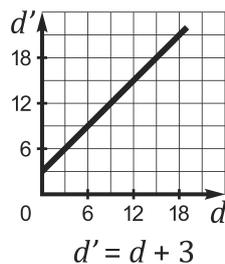


Рис. 5

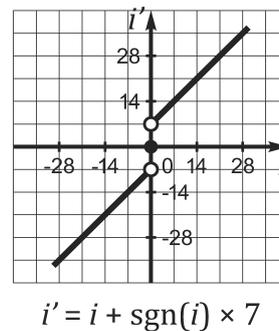


Рис. 6

Четвёртый канон. Пропоста — нижний голос. Длительности респосты выведены по формуле $d' = |-d^2 + 8d - 6|$ (рис. 7). «Неудобные» значения округлены. Например, $2\frac{8}{9}$, которое получается из триольной восьмой ($1\frac{1}{3}$), интерпретировано как восьмая с точкой (3).

В отношении высот респоста является обращением пропосты, как в первом каноне: $p' = -p + 5$.

Технически в Четвёртом каноне два канона, назовём их канон-А и канон-В. Второй расположен «внутри» первого. В пьесе три раздела. Канон-А прерывается в конце первого раздела и возобновляется в третьем. Канон-В звучит во втором разделе пьесы.

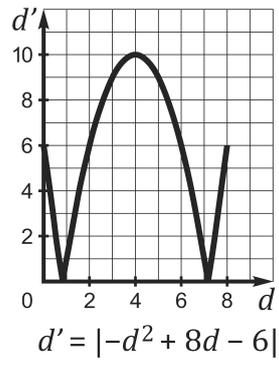


Рис. 7

* * *

Canon 1

Allegro moderato

Piano

The score consists of six systems of piano and bass staves. The piano part is marked with dynamics *p*, *f*, *mf*, *ff*, and *pp*. The bass part is marked with *f*, *mf*, *ff*, and *p*. The score includes various musical notations such as slurs, accents, and dynamic hairpins. Specific markings include *cresc molto* in the final system. The piece is in 2/4 time, as indicated by the first measure.

Musical score system 1, featuring a treble and bass clef. The treble clef contains a melodic line with various ornaments and dynamics: *mf*, *p*, *mf*, *pp*, *mf*, *pp*, *mf*. It includes triplets and a 2nd measure rest. An *ossia* bracket indicates an alternative ending with a *p* dynamic. The bass clef provides a harmonic accompaniment.

Musical score system 2, treble clef. Dynamics include *cresc molto*, *ff*, and *mf*. It features a triplet, a 5-measure rest, and a 6-measure rest. The tempo marking *tempo* is present.

Musical score system 3, treble and bass clefs. Dynamics include *f* and *p*. It contains a 3-measure rest and a 5-measure rest.

Musical score system 4, treble and bass clefs. Dynamics include *ff*. It features a 7-measure rest.

Musical score system 5, treble and bass clefs. Dynamics include *mf*, *p*, *f*, *f dim*, and *mf*. It contains multiple triplet markings and a 3-measure rest.

Musical score system 6, treble and bass clefs. Dynamics include *mf*, *cresc*, *f*, *p*, *cresc*, and *f*. It features a 2-measure rest and a 3-measure rest.

Musical score system 7, treble and bass clefs. Dynamics include *mf*, *f*, *ff*, and *f*. It contains a 3-measure rest and a 2-measure rest.

Canon 2

Presto

The musical score for Canon 2 is presented in a grand staff format, consisting of a treble clef staff and a bass clef staff. The piece begins with a **Presto** tempo marking. The first system includes dynamics of *f*, *capriccioso*, *mf*, *ff*, and *mf*, with articulation marks such as accents and slurs. The second system continues with dynamics of *f*, *mf*, *p*, *f*, *mf*, and *mf*. The third system features *p*, *mf*, and a dynamic swell. The fourth system includes *f*, *p*, *mf*, *p*, and *f*. The fifth system shows *mf*, *ff*, and *mf*. The sixth system is marked **poco meno mosso** and includes dynamics of *f*, *dim*, and *p*. The seventh system is marked **tempo** and includes dynamics of *f*, *dim*, and *p*. The score is filled with various rhythmic values (e.g., 3, 2, 5, 8, 4, 3, 2, 4, 5, 2, 3, 5, 5, 2+5+2), slurs, and articulation marks. The key signature has one sharp (F#).

2♩ 2♩+♩ 4♩ 8.....
f p f p *ad lib*

3♩ 3♩ 5♩ 3 4♩ 3♩
f p ff pp f p mf f mf p

quasi accel.....
 5♩ 3 5 3 5 3
cresc f ff

tempo
 2♩+♩ 4♩ 8..... 3♩
p f p p 3 p

2♩ 2♩ 4♩ 4x5♩
f mf

poco meno mosso tempo
 2♩ ♩+2♩ 5♩ 3♩ 5♩+♩ 5♩ 2♩+5♩+2♩
f dim p

Canon 3

Adagio

4/4

The musical score for Canon 3, Adagio, is written in 4/4 time. It consists of three systems of two staves each (treble and bass clef). The first system begins with a treble staff containing a whole rest and a bass staff starting with a forte (*f*) dynamic. The second system continues with a piano (*p*) dynamic in the treble and a mezzo-forte (*mf*) dynamic in the bass. The third system concludes with a mezzo-forte (*mf*) dynamic in the treble and a forte (*f*) dynamic in the bass. The score includes various musical notations such as notes, rests, slurs, and dynamic markings.

4.

p

f *p* *mf* *p*

pp dolce

mf

p

f *p* *mf* *f* *mf*

Canon 4

Allegro moderato

The musical score for Canon 4 is presented in two systems, each with a treble and bass staff. The piece is in 3/4 time and features a variety of dynamics and articulations. The first system begins with a treble staff containing a whole note rest, followed by a half note, and a measure with a fermata. The bass staff starts with a forte (*f*) chord and continues with a rhythmic pattern. The second system features a treble staff with a half note, a whole note, and a series of eighth notes. The bass staff includes a mezzo-forte (*mf*) chord, a triplet of eighth notes, and a triplet of quarter notes. The third system shows a treble staff with a series of eighth notes and a forte (*f*) chord. The bass staff includes a piano (*p*) chord and a mezzo-forte (*mf*) chord. The fourth system features a treble staff with a triplet of eighth notes, a mezzo-forte (*mf*) chord, and a piano (*p*) chord. The bass staff includes a pianissimo (*pp*) chord and a piano (*p*) chord. The fifth system shows a treble staff with a half note, a quarter note, a half note, and a quarter note. The bass staff includes a forte (*f*) chord and a piano (*p*) chord.

System 1: Treble clef, bass clef. Treble staff contains a melodic line with slurs and accents. Bass staff contains a rhythmic accompaniment. Dynamics include *cresc* and *ff*. A triplet of eighth notes is marked with a '3' above it.

System 2: Treble clef, bass clef. Treble staff features a melodic line with slurs and accents. Bass staff features a rhythmic accompaniment. Dynamics include *mf* and *p*. A triplet of eighth notes is marked with a '3' above it.

System 3: Treble clef, bass clef. Treble staff features a melodic line with slurs and accents. Bass staff features a rhythmic accompaniment. Dynamics include *f* and *p*. A triplet of eighth notes is marked with a '3' above it.

System 4: Treble clef, bass clef. Treble staff features a melodic line with slurs and accents. Bass staff features a rhythmic accompaniment. Dynamics include *p*, *f*, *mf*, and *cresc*. A triplet of eighth notes is marked with a '3' above it.

System 5: Treble clef, bass clef. Treble staff features a melodic line with slurs and accents. Bass staff features a rhythmic accompaniment. Dynamics include *ff*. A triplet of eighth notes is marked with a '3' above it.

System 1: Treble clef, two whole notes, then a rest. Bass clef, eighth notes, quarter notes, and eighth notes. Dynamics: *mf*.

System 2: Treble clef, eighth notes, quarter notes, eighth notes, quarter notes, eighth notes, quarter notes. Bass clef, eighth notes, quarter notes, eighth notes, quarter notes, eighth notes, quarter notes. Dynamics: *p*. Includes a triplet in the bass clef.

System 3: Treble clef, quarter notes, quarter notes, quarter notes, quarter notes, quarter notes, quarter notes. Bass clef, eighth notes, quarter notes, eighth notes, quarter notes, eighth notes, quarter notes. Dynamics: *mf*, *pp*, *p*. Includes a triplet in the bass clef.

System 4: Treble clef, eighth notes, quarter notes, eighth notes, quarter notes, eighth notes, quarter notes. Bass clef, eighth notes, quarter notes, eighth notes, quarter notes, eighth notes, quarter notes. Dynamics: *pp*, *p*. Includes a triplet in the treble clef.

System 5: Treble clef, quarter notes, quarter notes, quarter notes, quarter notes, quarter notes, quarter notes. Bass clef, eighth notes, quarter notes, eighth notes, quarter notes, eighth notes, quarter notes. Dynamics: *mf*.

System 6: Treble clef, quarter notes, quarter notes, quarter notes, quarter notes, quarter notes, quarter notes. Bass clef, eighth notes, quarter notes, eighth notes, quarter notes, eighth notes, quarter notes. Dynamics: *f*, *p*, *f*, *p*. Includes a triplet in the treble clef.

Musical score for Sergei Zagny's *Four Canons*, page 11. The score is written for two staves (treble and bass clef) and includes various musical notations and dynamic markings.

The score is divided into seven systems, each with two staves. The dynamics and markings are as follows:

- System 1: *mf* (both staves)
- System 2: Treble staff has *mf*, *ff*, *mf*; Bass staff has *ff*, *mf*
- System 3: Treble staff has *f*, *p*, *f*; Bass staff has *p*, *f*, *p*, *f*
- System 4: Treble staff has *mf*, *f*, *mf*; Bass staff has *p*, *f*, *p*, *f*, *mf*, *p*
- System 5: Treble staff has *ff*, *mf*, *p*; Bass staff has *f*, *mf*, *f*, *p*, *mf*
- System 6: Treble staff has *cresc*, *f*; Bass staff has *p*, *mf*, *f*

The image displays a musical score for Sergei Zagny's "Four Canons", page 12. The score is written for piano and bass, consisting of three systems of two staves each. The key signature is G major (one sharp) and the time signature is 3/4. The first system begins with a piano (*p*) dynamic in the right hand and a piano (*p*) dynamic in the left hand. The second system features a forte (*f*) dynamic in the right hand and a mezzo-forte (*mf*) dynamic in the left hand. The third system starts with a piano (*p*) dynamic in the right hand and a piano (*p*) dynamic in the left hand. The score includes various musical notations such as slurs, accents, and dynamic markings.

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