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A Genealogic-Structural Hypotesis of Harmonic Dualism

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Abstract: In "classical" music theory, the minor triad has always been considered subordinate to the prevailing, and more "natural", major triad (both for major and minor tonalities). Harmonic dualism is a principle that ascribes equal importance to the major and minor triads approached from dialectical and acoustical point of view, i.e. in terms of undertones and overtones, mediation of contrasting forces and physiological-structural understanding (following Oettingen's (1866), Hauptmann's (1853), and Riemann's (1905 and 1915) studies). However, these studies do not seem to convincingly justify the existence of the minor triad; all attempts to trace the foundation of the minor triads in Oettingen's series of undertones appears equally weak. Being the minor triad — and tonality — an indisputable piece of evidence, the David Lewin's *A Formal Theory of Generalized Tonal Functions* (1982) is the only one able to demonstrate the structural-formal explanation of harmonic dualism in terms of inversion of the same intervals which belong to the major triad.

Considering that the tonal praxis is likely to come from transposition of psalm-tones in the instrumental music of the XVII century (following Barnett's (1998), and Powers's (2014) papers), we seek the ontological basis of harmonic dualism in the praxis of temperaments; depending on the higher or lower consonance of the two triads in meantone temperaments (for simplicity 1/3 and 1/4 of syntonic comma (s.c.) meantone temperaments), we can trace back a double dualism, in which the third in question is ontologically prevalent, and the complementary one is its dual. However, temperaments do not allow modulations without a transposition to distant keys. Conversely, Lewin's model allows generating "dual" triads by mathematical manipulation of their constituents — i.e., inversion ("TDINV" operation) and/or conjugation ("CONJ" operation). By looking at the modal diatonic sets on which psalmodic-tones are based (the natural notes with added B♭) and by arranging the pitches of these tones as thirds, one can obtain two palindromes which represent all the triadic possibilities for these modal sets, which confirms the structure theorized by Lewin (which is a palindrome as well).

Our approach contradicts the theories of "classical" harmonic dualism, which focuses only on the single constituents of the triads (regarding the topics cited above). Moreover, such approaches seek to identify how the model could be adapted to the temperament independently, i. e. without including the temperament itself into the model. All these aspects fail to provide a convincing explanation of the harmonic dualism, that wasn't achieved until Lewin's formulation. By considering chords as autonomous entities, different from the ordinary superposition of scalar intervals, Lewin provided for the first time a formally convincing explanation based on the equal temperament, i.e. without making comparable consonance and "naturalness" of the major and minor triad a consequence of a specific intonation (following Lewin's 1982 paper).

My paper aims at highlighting the relevance of a historically informed genealogy of the harmonic dualism, in relation with the musical practices that led to the emergence of tonality through the use of temperaments and their interval structure.

Keywords: harmonic dualism, Major/Minor triads, modality, tonality, temperament

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Научная статья

Генеалогико-структурная гипотеза гармонического дуализма

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Аннотация: В «классической» теории музыки минорное трезвучие всегда считалось подчиненным господствующему и более «естественному» мажорному (как в мажорных, так и в минорных тональностях). Принцип гармонического дуализма придает равную значимость мажорным и минорным трезвучиям, рассматриваемым с диалектической и акустической точек зрения, т. е. с точки зрения унтертонов и обертонов, опосредования противоположных сил и физиолого-структурного понимания (см. труды Эттингена (1866), Гауптмана (1853) и Римана (1905 и 1915)). Однако оправдание существования минорного трезвучия, предлагаемое в этих трудах, не выглядит убедительным; все попытки обнаружить основание минорных трезвучий в унтертоновых рядах Эттингена оказываются в равной мере безуспешными. Хотя существование минорного трезвучия — и минорной тональности — не подлежит сомнению, лишь «Формальная теория обобщенных тональных функций» Дэвида Льюина (1982) способна дать структурно-формальное объяснение гармонического дуализма, исходя из инверсии интервалов, составляющих мажорное трезвучие.

Принимая во внимание тот факт, что тональная практика, по всей вероятности, берет свое начало в транспозиции псалмовых тонов в инструментальной музыке XVII века (см. публикации Барнетта (1998) и Пауэрса (2014)), мы ищем онтологическую основу гармонического дуализма в практике темперации; в зависимости от взаимного расположения консонансов двух трезвучий в среднетоновых темперациях (для простоты назовем их среднетоновыми темперациями на 1/3 и на 1/4 синтонической коммы) можно проследить двойной дуализм, в котором обсуждаемая терция онтологически преобладает, а дополняющая является ее двойником. Однако темперации не позволяют осуществлять модуляции без транспозиции в отдаленные тональности. И наоборот, модель Льюина позволяет генерировать «двойственные» трезвучия путем математических манипуляций с их составляющими — инверсии (операция «TDINV») и/или сопряжения (операция «CONJ»). Если посмотреть на модальные диатонические звукоряды, на которых основаны псалмовые тоны («белые ноты» с добавлением си бемоля), и расположить звуковысоты этих тонов по терциям, можно получить два палиндрома, репрезентирующие все возможные в рамках данных модальных звукорядов трезвучия; это подтверждает структуру, которую Льюин получил теоретическим путем (и которая также является палиндромом).

Наш подход находится в противоречии с теориями «классического» гармонического дуализма, при котором внимание фокусируется только на одиночных составляющих трезвучий (в названном выше контексте). Более того, такие подходы стремятся определить, каким образом можно приспособить модель к темперации независимо от таковой, т. е. не включая в модель саму темперацию. Все эти аспекты не позволяют дать убедительное объяснение гармоническому дуализму, и такое положение вещей существовало до тех пор, пока Льюин не сформулировал свою теорию. Отойдя от обычной практики и рассматривая аккорды как автономные сущности, а не наложения принадлежащих гамме интервалов, Льюин впервые дал убедительное с формальной точки зрения объяснение, основанное на равномерной темперации, не выводя сопоставляемый консонанс и «естественность» мажорного и минорного трезвучий из особой интонации (см. упомянутую статью Льюина).

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Цель моей статьи — подчеркнуть значимость исторически информированной генеалогии гармонического дуализма, рассмотрения его с точки зрения музыкальных практик, которые привели к возникновению тональности благодаря использованию темпераций и их интервальной структуры.

Ключевые слова: гармонический дуализм, мажорное и минорное трезвучия, тональность, модальность, темперация

I. Introduction

Regarding harmonic dualism, there are some theories based on many dialectic and physical phenomena, yet none of them clarify its real structure at all. Moritz Hauptmann, Artur von Oettingen and Hugo Riemann tried to demonstrate the ontological harmonic dualism's structure respectively on the dialectical, physical and physiological-psychological fields. In 1982 David Lewin found a mathematical-formal schema of tonal syntax which demonstrated the dualism inside of triadic languages. This schema is similar to the manner in which temperaments are built in their disposition of pitches' intervals, so it is possible to find a first manifestation of that in the temperaments adopted in psalm-tones, which formed modal triads in counterpoint, used at the beginning of the first tonal musical language's manifestations.

II. Exposition — "Classical" Harmonic Dualism's Theories (Hauptmann, Oettingen and Riemann) and new perspectives

In *Die Natur der Harmonik und der Metrik* (1853) Moritz Hauptmann introduced the dualistic view of harmony. Hauptmann identified the harmonic "nature" in Subdominat-Tonic-Dominant (S–T–D) triads' relation, the so called "Triad of triads". Those triads were formed from the tonal root in the major tonality, and from the tonal fifth in minor one; both of these pitches were identified with "I", because those are respectively the *first* of the major and minor triads, and have a fifth interval above or below (identified with "II") and a major third above or below ("III"), so the minor triads are building with the same intervals of the major one, but disposed from higher pitch to lower.

Hauptmann give a dialectical explanation of this "Triad of triads" which was the same explanation of the dialectics applied to every triad's pitches: the *first* "I" (tonal root in major triad and the fifth in minor one) which corresponds to "state of unity", the "II" (a fifth above or below) which corresponds to "state of opposition to unity", and the "III" which corresponds to "state of joining together of unity and opposition" [13, *52*; 22, *69*; 10, *94–95*]. Hauptmann's explanation is not rooted in physical proof, it was only philosophical and dialectical: the harmonic mean is a dynamic complex of "forces" between the three dialectical meanings of triad's pitches. Kopp summarizes the phenomena in this manner: "The genesis of harmony is the result of a three-part process: (I) the ground-state of the individual chord; (II) the opposition of two chords in succession; and (III) their synthesis in harmonic relation." [13, *55*] (Example 1).

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In 1866 the physicist Artur von Oettingen developed Hauptmann's dualism in his *Harmoniesystem in dualer Entwiklung.*

Example 1.

a) Dialectical meanings in Hauptmann's Triad, Triads in fifth's relation and "Triad of Triads" [22, *69*];

b) minor tonality's "Triad of Triads"

a)

$$(I)$$
$$I - III - II$$
$$C \quad e \quad G$$

Legend

I = Unity
II = Opposition to Unity
III = Joining together of unity and opposition

$$(I) \qquad - \qquad (II)$$
$$I - III - II \qquad\qquad I - III - II$$
$$F \quad a \quad C \quad e \quad G \;/\; C \quad e \quad G \quad b \quad D$$
$$\qquad I - III - II \qquad\qquad I - III - II$$

$$(I) \qquad - \quad (III) \quad - \quad (II)$$
$$S \qquad\qquad T \qquad\qquad D$$
$$I - III - II \qquad I - III - II$$
$$F \quad a \quad C \quad e \quad G \quad b \quad D$$
$$\qquad I - III - II$$

b)
$$II - III - I \qquad II - III - I$$
$$d \quad F \quad a \quad C \quad e \quad G \quad b$$
$$\qquad II - III - I$$

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Following Helmoltz's acoustic studies, Oettingen achieved a system which formed the minor triads from overtones and the major triads from undertones.[1] The acoustic principles which founded this view of harmony were "phonicity" and "tonicity": phonicity is the property of a chord or an interval to be grasped of a partial of a fundamental, while tonicity is the property of a chord or an interval to be a set of common partials of a fundamental pitch (e.g.: major triad c_2–e_2–g_2 has the c_0 fundamental undertone and b_6 phonic overtone; minor triad c_2–eb_2–g_2 have ab_1 fundamental undertone and g_6 phonic overtone).

Oettingen's triadic structure was formed following this acoustic element (and corresponds with Hauptmann's one): the tonic fundamental of major triad is structural parallel of the phonic overtones of minor one, and these tones are consonant with their respective chords and dissonant with complementary chords.

This structural interpretation of triad's meaning was the point of departure for Riemann's dualistic theory of harmony, and more precisely for his "Step-Change" (*Schritt-Wechsel*) system, based on the assumption that the more intelligible transformation is *Seitenvechsel*, or the inversion of a triad from its *first* (i.e. the same logic of Oettingen's tonicity and phonicity). Furthermore Oettingen in his *Harmoniesystem* made a diagram of tonal space where the pitches were arranged in a relation of thirds and fifths, something very similar to Riemann's *Tonnetz* (Example 2) [14, *462–465*; 22, *22–26*].[2]

Riemann developed Hauptmann's and Oettingen's theories and improved harmonic dualism's theory, also in the direction of harmonic functions.

In *Das Problem des harmonishen Dualismus* (1905) Riemann underlined how its impossible overtones series are the reason of consonance: some overtones in major triads are both as dissonant as minor ones, and minor triads can't be consonant for the same reason. For Riemann the consonance's reason is in common overtones of major and minor triads, and there is a symmetric building structure in which major triads are formed upward from the undertone (tonic) and minor triads are formed downward from the overtone (phonic), in coherence with Oettingen's theories. Differently from Oettingen, for Riemann the consonance is no more a physical-acoustic phenomena, but a perceptive-acoustic evidence which is evident in the reciprocal relations of triads in hearing music, without any correlation to "natural" science. For these reasons the "root" [*Groundtone*] of a major triad is the tonic, and in a minor one the "root" became the fifth's interval.

These ideas were an improved version of some topics contents from *Die Natur der Harmonic* (1882). In this paper Riemann introduced the idea of a psychological process linked to the musical meanings of chords, derived from physical and acoustical fields [23, *65*].

Riemann justified undertones' existence with the Zarlino's theories on a triadic structure, the theories of thorough bass — where, for example, a vii[7°] is a sort of V[9] without the root, and not a vii harmony — and the existence of Tartini's third sound found an evidence on

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[1] Helmoltz had disputed Hauptmann's harmonic dualism: he claimed the "inferior" nature of minor chords from of the major ones because they have interference among its upper partials. Oettingen, in other hand, declared the equal ontological status of major and minor chords, and showing theirs symmetric generating structure [14, *462–463*].

[2] It's important to underline how both Oettingen's diagram and Riemann's *Tonnetz* are two diagrams which starts from a pitch and applied some intervals to it; in the aim of this paper it's important point out this because they are two kind of temperaments.

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a dualistic nature of harmony in which "the major and minor consonances are, according to their mathematical-physical relations, strict opposite of one another" [23, *78*].[3]

Following on from these topics, Riemann's triadic structure was derived from the vibrating string length for the minor chords, and from frequency proportion for the major. The concept is confirmed and extended in the field of harmonic function, also in the introduction of 1895's *Harmony Simplified or The Theory of the Tonal Functions of Chords* [24, *1–9*], with added definitions of theories of melody and harmony in which harmony is "*correct connection of chords*" arisen "*from simultaneous melodic motion of several parts*" [ibid., *1*] derived by development of polyphony in history of music, and "Melody *is the logically rational and aesthetically satisfactory motion of a part through notes of different pitch*" [ibid.].

The harmonic functions are only three: S (subdominant) or °S (minor subdominant), T (tonic) or °T (minor tonic) and D (dominant) or °D (minor dominant), and the two tonal systems are S–T–D in the major system and °S–°T–°D in the minor system. In the major system S may be exchanged with °S, whereas in the minor system °D can be replaced with D; in those cases the "mixed system" is obtained [15, *199–205*; 24, *44–49*].

The theory of functions in *Harmony simplified* and the ideas on the musical perception cited above was improved by Riemann in his 1915's *Ideen zu einer 'Lehere von den Tonvorstellungen'*. In this paper Riemann developed the idea of tones as imagined in reciprocal distance following cultural knowledge, so the human perception is strictly linked to behavior derived from cultural rules and the ability of the brain to *immagine the tones* [*Tonvorstellungen*]. In our Western cultural behavior, according with Riemann, we can imagine the tones following major thirds and fifths; for this reason he builds his "three-dimensional" *Tonnetz:* a topographical representation of pitches arranged by

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[3] Zarlino was the first theorist who theorized the inversion of intervals of major triad for building the minor one, so the minor triad has the same intervals of major but in an inverted or mirrored disposition: the major third is beneath the minor in major triad and above the minor in the minor triad. Riemann for this topic considered Zarlino as dualist, but he was wrong because Zarlino never theorized an inversion of *harmony in triads*, but a different position of intervals derived from *voice-leading in counterpoint* [14, *462*; 4, *289*; 5, *234–238*], and Riemann's mistake was probably a consequence of a translation misunderstanding [4; 22, *26–31*]. Furthermore Zarlino considered major triads "natural" and minor one "accidental" [5, *111–114*]), whereas harmonic dualism considers both triads as equivalent, "derived from a single, unitary process that structurally contains the potential for twofold, or binary, articulation" [14, *459*]); "Major and minor triads come about as two distinct but equal expressions of a single structuring principle, and, as such, together from the whole or totality of harmonic possibilities." [15, *196*]). On this disagreement, Alexander Rehding takes a position which seems definitive: "The implicit assumption on both Riemann's and Dahlhaus's parts, that Zarlino must be either dualist or a monist, is pure music-theoretical ideology." [22, *30*]). However Zarlino's claims had some elements which disclosed the harmonic dualism's methodological approach: e.g., despite the differences of concepts — in Zarlino's view intervals are arithmetic proportions and triads are intervals piled one another –, the subdivision of the fifth in two thirds for building triads, and the intervals considered upper and lower from a pitch (i.e. *Subdupla-dupla* [upper and lower eighth], *subsequialtera-sesquialtera* [upper and lower fifth], *subsesquitertia-sesquitertia* [upper and lower forth], etc.) [4, *298*; 5, *234–238*]. Furthermore Zarlino's challenge for legitimate minors triads are an evident sign of his purpose to bring them in coeval theory.

Respect the undertones and their correlation with Tartini's third sound, there is a physical study by Michelangelo Abbado made in Milano's Politecnico, Physics Department, where he found some undertones when he played the violin in some particular manner. Unlike this study had no successive detailed studies, so it's not enough to prove the existence of undertones — although Tartini's third sound is a well known phenomena [1, *234–237*]).

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fifths in west-east direction and for major thirds in south/west-north/east directions, so the *Tonnetz* also serves as a mental map in which every chord relation has a mental meaning linked to functions, and this map is a temperament based on pure major thirds and fifths.[4]

These proofs on the existence of harmonic dualism are not so persuasive, but the existence of both major and minor tonality is an indisputable evidence. So the questions are: what is tonality? And where does it comes from? Are there any aspects of harmonic dualism in the elements which foreshadow tonality?

If the answer of first question is "syntax of triadic relation's meaning" and the answer of second is "from modal polyphony in Renaissance music", then the answer of third question may be found in the use of triads derived from Renaissance's counterpoint (modal triads), in their triadic "potential" (i.e. the triads or tonal functions which were permitted and/or avoided) and in the structural praxis which placed pitches' sizes in triads derived from counterpoint (i.e. the intonation and/or temperaments adopted and how the pitches were disposed to form modal triads) [6].[5]

The passage from modal to tonal praxis took place during the Baroque era, when some composers started writing music with modal criteria but in a tonal way [2; 20], in which music perception was more tied to a harmonic series of events than melodic ones [11, *738–739*]. In this period the tonality was a transposition of a "core" of psalm-tones, or church-keys, in every pitch. This "core" enclosed Riemannian's S–T–D and °S–°T–°D functions, respected Lewin's formal theory and adopted historic temperaments — e.g. 1/4 and 1/3 s.c. mean-tone temperaments which were adopted in Renaissance and Baroque ages — which allow a sufficient consonance in these harmonies [6, *53–60*, *78–85*]. The harmonies' first appearance was in Renaissance polyphonic music's *modal triads*, which were the triads made in modal criteria by the voices in counterpoint in the ending modal *clausulae* [2; 19–21].

This paper's purpose is to search the correlations between harmonic dualism and the preexisting aspects of tonality and try to seek a direct correlation between modal triad's pitches, harmonic dualism and historic temperaments — for the sake of simplicity 1/4 of s.c., for major thirds, and 1/3 of s.c. mean-tone temperaments for minor thirds — for a genealogical hypothesis of harmonic dualism's birth.

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[4] It's difficult to summarize Riemann's supposal shortly, because he was very busy to improve his theories and ideas, which changed in his life course. Here I tried to show the more important and pertinent element of harmonic dualsm's theory in my paper's direction.

[5] Regards "tonality", Brian Hyer give this definition: "As a music-theoretical term, 'tonality' was first used by Alexander Choron in 1810 to describe the arrangement of the dominant and subdominant above and below the tonic and thus to differentiate the harmonic organization of modern music (*tonalité moderne*) from that of earlier music (*tonalité antique*). One of the main conceptual categories in Western musical thought, tonality most often refers to the orientation of melodies and harmonies toward a referential (or tonic) pitch class. In the broadest possible sense, however, it refers to systematic arrangement of pitch phenomena and relations between them." [11, *726*].

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III. Elaboration — Theoretical problems on Harmonic-dualism's theories and David Lewin's formal solution

There are some problems in the dualistic theories cited above. The more evident ones are: the dialectical nonsense of Hauptmann's system where the dialectic tonality's means of union ("I") is the Subdominant triad and not the Tonic one (which is a sort of "reconciliation between opposite forces" — "III"); the theory of undertones are too weak to justify the minor triad's downward structure.

Moreover, in a 1984 paper, David Lewin noticed that Riemann's functions are not isomorphic with scale-degree space if in Wagnerian excerpts we identify an E♭♭ in the same time as a sort of A♭'s D and S function: in Lewin's claims, starting from A♭ "six plagal cadences piled consecutively one of another execute a 'dominant' function", and therefore made an E♭♭ pitch, but also "the Neapolitan of the subdominant, that the harmony 'is' E♭♭[…]; 'therefore' the Neapolitan of the subdominant has a dominant effect. / The false syllogism demonstrate an important theoretical point. *The nature and logic of Riemannian tonal space are not isomorphic with the nature and the logic of scale-degree space*" [17, *345*].

This quote opens a new point of discussion: the correlation between the tuning and the harmonic objects.

Furthermore Lewin is not so exact: Riemann relates Neapolitan chord to church modes, so a Neapolitan must be a diatonic half step distance from his pitch (the Neapolitan of E♭ is D, not E♭♭) [24, *88–106*]; this is a purely theoretical point since the tonal space is in an equal temperament context, but this is no longer the case if the space became a different tuning. Riemann himself declares necessity of more studies in enharmonic field for the "enharmonic equivalent tone-values" — i.e. the pitches like E♭♭ and D — because he can't describe the role of that pitches in *Tonvorstellungen*, and making that he recognized the correlation between his system and the intonation: Riemann claims that *imagination of tone* is linked by pure tuning and the tempered scale is a compromise in intonation which allows us to use the tonal space at all, but in a better understanding in the central area of tonal system; so Riemann express his conviction that some later studies "*of enharmonic identification*" may solve the problems of "*the exchange of coinciding enharmonic tone-values*" [26, *109–110*].

Riemann's *Tonnetz* has a more consonant region near the central starting C pitch. The pitches from this level have some lines above or below to show how many thirds intervals there are from central pitches, and there are equivalent enharmonic pitches on the edges of *Tonnetz* which are the more distant from central pitches and less consonant with them (Example 2) [ibid., *102*].

This point of view opens a wider problem embracing the music theories of harmony at all: the difference from the tonal system and tuning, which was declared by Carl Dahlhaus in his *Studies on the Origin of Harmonic Tonality*: for Dahlhaus this lack of knowledge generated some misunderstanding because it generates "a confusion of concepts that comes dramatically to light the habit of speaking of 'tonal system' and meaning instead 'tunings'" [5, *187*]. Following this topic, Dahlhaus achieved a conclusion similar to Riemann's one, in which there is a misunderstanding in theory that does not consider the difference because it

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was never examined [ibid., *191*]. Another example of this problem is the "meaning of major third", which may be understood in three ways: a *Durterz* which corresponds to a major third in "just intonation" (4:3 proportion), a ditone which corresponds to two whole tones, and a distance of 4 half-steps conforms to the logarithms of equal temperament. These three meanings of "major third" made three different musical objects, and may create big theoretical misunderstandings [ibid., *61–63*].[6]

Example 2.   Riemann's *Tonnetz* [26, *102*]
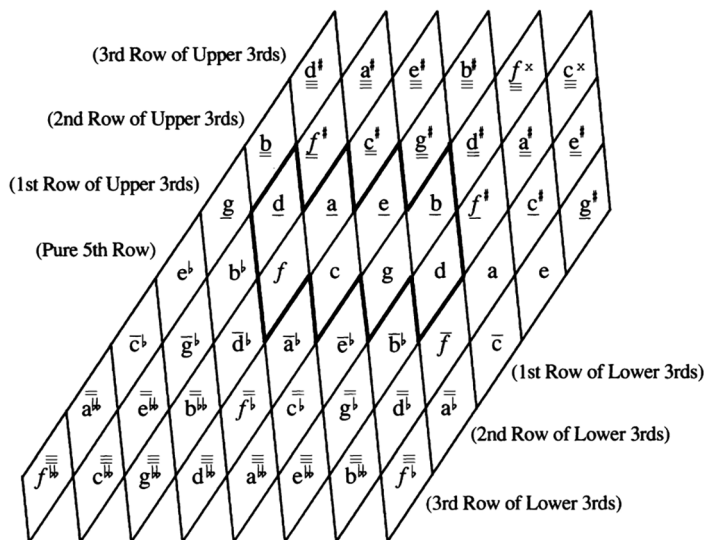

In 1982, David Lewin achieved a mathematical-empirical explanation of harmonic dualism in terms of inversion and/or conjugation of the same intervals of major triads. Lewin created a "Riemann System" — so called in Hugo Riemann's memory — in which he applied some half-step intervals: an **m** interval (mediant interval) and a **d** interval (dominant interval) to a **T** pitch, and the difference **d-m** made **m'**, so he formed a triad. From this structure he found two ways to obtain the opposite triad: the inversion of the **T** pitch and the dominant interval **d** ("TDINV" operation) and/or the substitution of **m** with **m'** interval ("CONJ" operation). Transpositions by **d** and **-d** of this triad made respectively the triads D or °D and S or °S (Example 3) [16, *23–36*].

For a "classical" consonant triad the **m** and **d** intervals correspond respectively to a third and a fifth, but it's not a necessary condition of existence for the system since **m+m'=d** and **(T+m)-m=T** [ibid., *25*].

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[6] Those differences in ancients ages was better known as an ontological topic of harmony and was the principal cause of temperament's necessity. Zarlino in his *Istitutioni Harmoniche* (Venice 1558, bk. 3, chap. 29) defined harmony as something can made only with diverse and discordant things: "Because they well knew that harmony can arise only from thing diverse, discordant, and contrary among themselves, and not from those things that agree in every aspects" [5, *21–22*].

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Example 3.   Formal structure of triads **m+m'** (ORIGINAL), **m'+m** (CONJUGATE) and its retrograde (TDINV), according with Lewin [16, *32*]
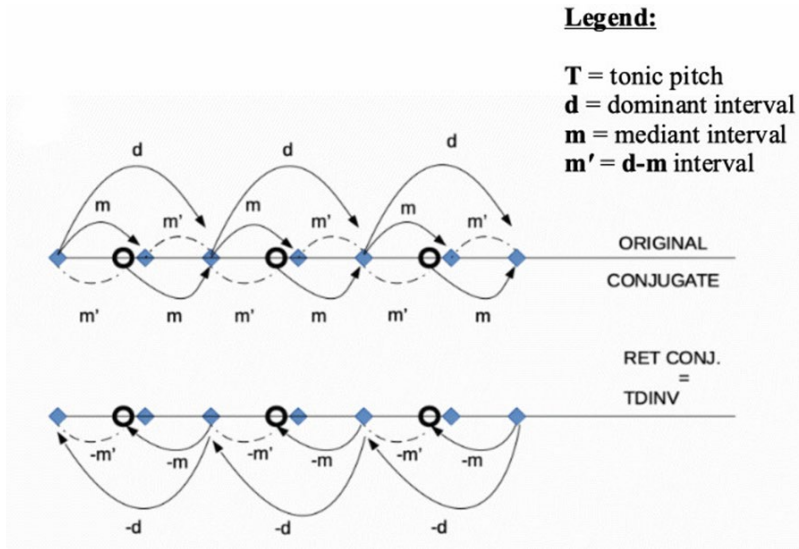
In this way Lewin demonstrated the isomorphism between triadic structure and triadic relations and at the same times proved the formal structure of harmonic dualism, which implies different triadic relations for major and minor genus as Riemann previously argued. It's also important to underline how this formal structure, with intervals applied to a pitch, is the same as that of a temperament. Lewin adopted equal temperament in this proof, but he claimed that it's "only a notational convenience here", and other sizes of **d** and **m** intervals are allowed [16, *60*].[7]

Lewin's system needs a palindrome placement of pitches' intervals and creates a symmetry which is very important in harmony and had an important role in the history of temperaments. The triad itself is a division of a fifth in two thirds, one major and another minor, which corresponds respectively to the arithmetic-mean and harmonic-mean of the fifth's proportion [9, *37–41*]; this aspect forced theorists to search some temperaments which allows this symmetry, in a historical development between irregular "well" temperaments and equal temperaments.[8]

It's evident how the temperaments are not only a "quality of sound" in music, but rather a structural element of pitches' configuration which influences the theoretical aspect of tonality. As regards to harmonic dualism and triadic syntax, it's necessary to examine the triadic combination possibilities which foreshadow tonality syntax

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[7] David Lewin formalized a "musical space" (a particular mathematical device to study specific textural feature of elements of a musical set) equivalent to Riemann's *Tonnetz* based on just intonation, but which can be reduced in an infinite series of 1/4 s.c. mean-tone temperament pitch classes [18, *17–22*].

[8] This topic is wide debated by Stuart Isacoff [12] and Kyle Gann [9]. For a brief dissertation on the differences of temperament's thought between Duffin [8], Isacoff and Gann, see [9, *162–163*].

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in Renaissance polyphonic music and the correlation between that and temperaments adopted in this practice (mean-tone temperament for the sake of simplicity and following Lewin's model).

IV. Confirmation — Development of Lewin's formal schema including temperaments and triad's history: a new perspective

All the theoretical devices presented here — Hauptmann's "Triad of triads", Oettingen's and Riemann's *Tonnetze*, and Lewin's formal structure — are based on a sequence of intervals which can be enclosed in temperaments: specific intonation of "Triad of triads" is "just intonation"; Oettingen's and Riemann's *Tonnetze* are both "two-dimensional" temperaments based on limit 5 — i.e. a temperament made on two specific intervals: major thirds' (5:4) and a pure fifths' (3:2) — because they are based on those intervals.[9]

These intervals' configuration are problematic because they do not allow a correct use of pitch-classes distant from the starting pitch (neither does Lewin's in a non-equal temperament). In Riemann's *Tonnetz*, C major has the problem of the D minor chord, which may be both upper fifth from G and lower fifth from A; in Hauptmann's "just" tuned "Triad of triads" it is narrowed by a s.c. [26, *99–101*; 22, *115–117*]. In "just" intonation C–D major whole tone has a proportion 8:9 and D–E minor whole tone has 9:10; for this reason the fifth D–A is narrowed by a s.c., and the solution adopted by Hauptmann was to switch these two whole tones: so he obtained his "minor tuning", which allowed °S–°T–°D triads to be consonant (and has the G–D fifth narrowed by a s.c.) [5, *191*].

It's clear how *Tonsystem* and *Tuning* are tied each others. The questions are how the tonality was born, what triads were used to make tonal syntax, and how the temperaments adopted allowed this process. The answers to these questions are in the praxis which made tonality. Riemann himself knew it when, in his *Harmony simplified*, he wrote the definitions of harmony and melody cited above [24, *1*].

Following Barnett's studies (1998, 2008), tonality was born during the 17th century when various composers began to write their music with modal criteria, adopted 8 "tonalities" derived from church-keys which Mattheson listed in his *Das Neu-eroffnete Orchestre* (1713), and later he transposed into all the 24 tonalities [3, *426–427*]. These 8 church-keys, in turn, adopted the modal triads derived from the specific modal ending-*clausolae*, and those are: D min. (1st Tone), G min. (2nd Tone), A min. (3rd Tone), E min. (4th tone), C Maj. (5th Tone), F Maj. (6th Tone), D Maj. (7th Tone) and G Maj. (8th Tone) [3, *427*; 5, *246–247*].[10] These modal triads may be thought of as the first manifestation of C Maj. and A min. tonalities, and have all the functional triads in those tonalities: F, C and G Maj.
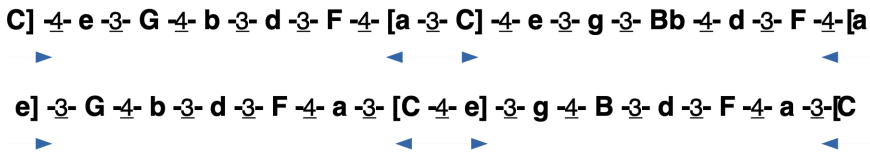
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9   Kyle Gann [9, *56*] shows an example of "just intonation" tuning which is the same as Riemann's *Tonnetz*. For more information about "just intonation" and five limits tuning the reader can see chapt. 5 of Kyle Gann, *The Arithmetic* [ibid, *55–62*].

10   The mode's development which created psalm-tones is complex and has a lot of misunderstandings derived from some differences between modal theory and it's application to first manifestations of tonality. This process moves around some theoretical explanation referred to modal criteria (which not was referred to harmonic phenomena, but tried to describe their's behaviors), the praxis of solmisation and the praxis to play the organ before a chant or between two chants for suggesting the right voices' intonation (*alternatim*). The result was a set of triads in the tones listed above which were used in a modal criteria, but foreshadow tonality [2; 3; 20].

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(6th, 5th and 8th Tones) as S–T–D in C Maj., and D, A and E min. (1st, 3rd and 4th Tones) as °S–°T–°D in A min. The fact that the origins of these psalm-tones are tied to the praxis of accompanying church chants with organ to give intonation to the choir (the so called *alternatim*), suggesting us that those triads must have a good consonance (otherwise they couldn't give correct intonation to the voices), so also the temperaments adopted in this praxis must to allow this [20, *positions 5944–6037*; 6, *56–60*].

The modal diatonic set is a diatonic scale with an added B♭, which was used in the praxis of solmisation. To understand how triads may be made with this set, we need to arrange pitches by thirds, and doing so obtain two palindrome sequences of intervals. The palindrome which starts with a minor third has both B and B♭, the major one has only B (see Example 4). These palindromes, which depict the triadic possibilities of modal diatonic sets, may be the first manifestation of Lewin's formal schema's logic.
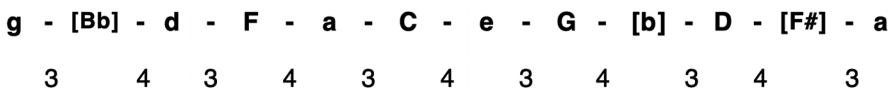
Example 4. Modal set's pitches arranged by thirds starting from minor and major thirds (numbers are intervals measured by half-steps)

C] -4- e -3- G -4- b -3- d -3- F -4- [a -3- C] -4- e -3- g -3- Bb -4- d -3- F -4-[a

e] -3- G -4- b -3- d -3- F -4- a -3- [C -4- e] -3- g -4- B -3- d -3- F -4- a -3-[C

Modal triads were strictly tied to modes, so they had an authentic version and a plagal version. The authentic one starts from the root-tone and the plagal from the fifth-tone as theorized by past theorists (e.g. by Banchieri, Kruger and Bononcini), so, for example, the modal triads of the 1st mode (D minor triad) is formed by D^2–F^2–A^2 in the authentic version and by A^1–D^2–F^2 in the plagal version [19, chapter 5]. It doesn't have the same structure of harmonic dualism, but it depicts how the triads were thought starting from 1ˆ or 5ˆ.

Later, when *musica ficta*'s praxis was consolidated and the modal triads had been transposed, this schema was extended and allowed the Lewin's one. In fact, the modal triads enclosed in Mattheson's psalm-tones were formed by these pitches arranged by thirds (which produced a palindrome which alternates minor and major thirds), and obtained the series of intervals depicted in Example 5 (in which F♯, B and B♭ are pitches of other triads enclosed in sequence, and aren't "roots" of triads). These schema transposed from the "core set" of psalm-tones in all tonalities essentially formed Lewin's.

Example 5. Mattheson's "Core" psalm-tones' pitches arranged by thirds

g - [Bb] - d - F - a - C - e - G - [b] - D - [F#] - a

   3   4   3   4   3   4   3   4   3   4   3

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Applying mean-tone temperaments to Lewin's formal structure, we achieve a schema in which we can obtain a specific triad by substituting **m** with the specific temperament interval: if, for simplicity, we take 1/3 of s.c. meantone temperament as minor thirds and 1/4 of s. c. mean-tone temperament as major thirds, calling the first **m** and the second **M**, we obtain a formal structure in which: with **m** we have a "natural" minor triad **T, T+m, T+d** (with **d** tempered at 1/3 of s.c.) and a complementary major triad **T, T+(d-m), T+d**; with **M** we have a "natural" major triad **T, T+M, T+d** and a complementary minor triad **T, T+(d-M), T+d** (with **d** tempered at 1/4 of s.c.). So, we can obtain two kinds of triads, which generated the triads of opposite genus with Lewin's CONJ and/or TDINV operations, as depicted in Example 6.

Example 6. Triads made with mean-tone temperaments applied to Lewin's formal shema.

Legend:
M= meantone 1/4 syntonic comma major third as Lewin's **m** interval
m= meantone 1/3 syntonic comma minor third as Lewin's **m** interval

MAJOR TRIADS

T-d	(T-d)+M	T	T+M	T+d	(T+d)+M	T+2d
F -	**a** -	**C** -	**e** -	**G** -	**b** -	**D**

T-d	T-m	T	T+(d-m)	T+d	(T+d)+(2d-m)	T+2d

T-d	T-M	T	T+(d-M)	T+d	(T+d)+(2d-M)	T+2d
F -	**ab** -	**C** -	**eb** -	**G** -	**bb** -	**D**

T-d	(T-d)+m	T	T+m	T+d	(T+d)+m	T+2d

MINOR TRIADS

The mechanism described in this process is the same of that of a temperament, where the only intervals available are applied above or below the starting pitch to obtain the pitches of the tempered scale. In this mode we have a "double dualism" in which both major and minor triads are "natural", and are also "accidental" since they derived from a TDINV and/or a CONJ operation, and the equality of major/minor genus wanted by harmonic dualism theories is achieved [6, *75–82*].

In this path, starting from the modal sets and their palindromes which foreshadow Lewin's, passing through the origins of tonality which adopted these sets and their transpositions, and arriving to Lewin's theory, there are two main topics: the triadic "potential" of diatonic sets and the temperament which determines the real sizes of intervals. In this genealogical view modal sets determine the pitches' disposition, while the temperaments were the first manifestation of Lewin's formal schema which allowed and conveyed triadic syntax.

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V. Conclusion

Following the argumentation above, if we consider harmonic dualism as a development of the structure imposed on pitches by diatonic modal sets and temperaments, we achieve a hypothesis on the harmonic dualism's genesis in which there are no more overtones and undertones, or dialectical meanings, but only a structure which was consolidated in the praxis of music throughout years of development in history of music. This hypothesis solves the classical problems of harmonic dualism by changing the perspective: harmonic dualism is a representation of a structure which describes harmonic relations following that relation's own development itself. Furthermore, this hypothesis made major and minor triads (and thirds) equally important and "natural", as derived by temperaments, and both major and minor triads are able to create the opposite triad's genus by rearranging its intervals, which harmonic dualism has been aimed to achieve since its first theorization.

This view of harmonic dualism's theories require a later study in enharmonic direction, as Riemann argued in his *Ideen*, and also the "double dualism" topic may be developed in some ways which allow triadic relations' analysis with both major and minor "dual" thirds, derived from the "complementary" genus of third.

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