



Research Article

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Extended Theory of Imitation and Canon, or New Methods of Transforming Proposta into Risposta

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Abstract: In strict imitations and canons, the *risposta* (consequent) may simply repeat the *proposta* (antecedent), but can also repeat it with some changes. In the generally accepted theory of imitation and canon the following transformations are admitted: augmentation-diminution, inversion, retrograde motion, and combinations thereof. We call these transformations *strict*, since for each there is a *rule* according to which the *risposta* can be derived from the given *proposta* unambiguously. However, there can be other strict rules. Hence, the methods of converting the *proposta* into the *risposta* are not limited to those listed above. What might these new methods be? In trying to answer this question generally and systematically, we use a mathematical concept of function. Function is a univocal correspondence between two sets. In our case function is a rule according to which the *risposta* can be derived from the *proposta* in a well-defined manner. Sets are melodies, the *proposta* and the *risposta*. The elements of a set are notes. A note in a melody is an object that has three dimensions: duration, pitch, and sequence number. At first we demonstrate conversions of durations into durations, pitches into pitches, and numbers into numbers. Always beginning with classical transformations (augmentation, inversion, retrograde motion) we then proceed to different methods. Then we consider “more exotic” conversions: durations into pitches, pitches into durations, etc. At the end of the article we attempt to demonstrate that some nonclassical transformations exist not only in our imagination, but also can be encountered in musical practice.

Keywords: imitation, canon, strict imitations, strict transformations of melodies, antecedent, consequent, *proposta*, *risposta*, duration, pitch, ordinal number

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Научная статья

Расширенная теория имитации и канона, или Новые способы превращения пропосты в респосту

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Аннотация: В строгих имитациях и канонах респоста может не просто повторять пропосту, но повторять ее с некоторыми изменениями. В современной теории имитации и канона допускаются следующие преобразования: увеличение-уменьшение, обращение, ракоход, а также их комбинации. Эти преобразования мы называем строгими, поскольку для каждого из них есть правило, согласно которому респоста выводится из данной пропосты однозначно. Но строгие правила могут быть и другими. А это значит, что способы превращения пропосты в респосту не ограничиваются теми, что были перечислены выше. Какими могут быть новые способы? Стараясь ответить на этот вопрос в общем и систематическом виде, мы используем математическое понятие функции. Функция — это однозначное соответствие между двумя множествами. Применительно к нашей теме, функция — это некое правило, согласно которому респоста выводится из данной пропосты единственно возможным образом. Множества — это мелодии: пропоста и респоста. Элементы множества — ноты. Нота в мелодии понимается как объект, у которого три измерения: длительность, высота и порядковый номер ноты в мелодии. Вначале рассматриваются превращения длительностей в длительности, высот в высоты, номеров в номера. Начиная каждый раз с классических преобразований (увеличение, обращение, ракоход), мы затем переходим к иным способам. Далее рассматриваются более «экзотические» превращения: длительностей в высоты, высот в длительности и т. д. В заключительной части статьи мы стараемся показать, что некоторые из неклассических преобразований существуют не только в нашем воображении, но и в музыкальной практике.

Ключевые слова: имитация, канон, строгие имитации, строгие преобразования мелодий, пропоста, респоста, длительность, высота, порядковый номер

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For imitations and canons, it is agreed that imitated and imitating voices (later in this text called *proposta* and *risposta*) are not necessarily the same. The following transformations are considered to be possible: (0) *exact repetition* (possibly with *transposition*); (1) *augmentation-diminution*; (2) *inversion*; (3) *retrograde motion*; and also the following combinations: (1)+(2), (1)+(3), (2)+(3) and (1)+(2)+(3). All these can be called *strict* transformations, since for each there is a *rule* according to which a *risposta* can be derived from a given *proposta* unambiguously.

However, there are many other transformations which are no less strict, but are not covered by the generally accepted theory to date. Some of these transformations were part of music practice, while some of them may have never been used before.

The aim of this article is to introduce the extended theory of imitation (or more exactly, its fundamentals¹). The extension concerns the region of the theory which studies the methods of transforming proposta into risposta.

1. DEFINITIONS

In this text we use the following definitions. *One-part structure* or *melody* (M) is some sequence of notes. A *note* (N) is an object determined by three values (parameters, dimensions). These are *duration* (d), *pitch* (p), and *sequence number* of a note in a melody (n). In other words, N is (d, p, n) . A *rest*, in contrast to a *sounding note*, is defined as a note with a *vacant pitch* (\emptyset), i. e. as (d, \emptyset, n) . If there are several successive rests in a music score, they are considered as a single rest, the duration of which is equal to their sum.

Original (imitated) *melody*, an object for transformation is the *proposta* (P). *Derivative* (imitating) *melody*, a result of transformation is the *risposta* (R). A note of the proposta is N or (d, p, n) . A note of the risposta is N' or (d', p', n') . An *imitation* is a number of melodies which correlate as proposta and risposta. The distribution of these melodies in parts (voices) and in time is of no importance.

To describe strict transformations, we use the mathematical concept of *function*. Function is a relation between two sets, a rule that associates each element of the first (independent) set with exactly one element of the second (dependent) set. Conventional notation is $y = f(x)$ (f is a rule of transforming x into y).

In our case independent and dependent sets are P and R. Independent and dependent elements are N and N'. A relation between them we express as $N' = f(N)$ or, considering the parameters of a note, as $(d', p', n') = f(d, p, n)$, where f is a rule of transformation. More complicated cases are also possible, when independent elements are two or more notes that belong to one or more propostas.

Hereafter for brevity, the expression *a duration of the proposta (risposta)* means "a duration of a note of the proposta (risposta)". Likewise, the expression *a pitch of the proposta (risposta)* is to be understood as "a pitch of a note of the proposta (risposta)".

2. TRANSFORMATIONS OF DURATIONS

Transformations of durations only, with other parameters remaining unaltered, can generally be defined as $(d', p, n) = f(d, p, n)$ or in short form as $d' = f(d, p, n)$.

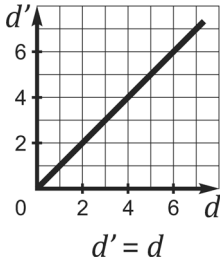
In this section we examine transformations where independent values are durations only: $d' = f(d)$.

Let us assign numerical values to durations. Let $\text{♪} = 1$, ♪^3 (triplet eighth) = $\frac{4}{3}$, $\text{♪} = 2$, $\text{♪}^3 = \frac{8}{3}$, $\text{♪} = 3$, $\text{♪} = 4$, etc. It is assumed that $\text{♪} = 0$.

¹ With many more details, but less accurately, this theory was stated in: [8]. I wrote also the following articles on this subject: [7; 9; 10].

2.1. Classical Transformations

Exact repetition. Durations of the proposta and the risposta remain the same. This is expressed by the formula $d' = d$.



$d: 6$ 2 4 2 2 4
 P:

 ↓ ↓ ↓ ↓ ↓
 R:

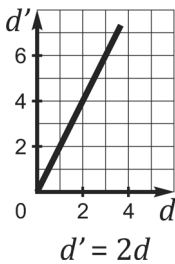
 $d': 6$ 2 4 2 2 4

In musical examples the upper line is a proposta, and the lower line is a risposta. Arrows connect the related notes.

Augmentation-diminution. The general formula for these transformations is $d' = ad$ ($a > 0$). $a > 1$ for augmentation; $0 < a < 1$ for diminution. (For exact repetition $a = 1$.)

Examples.

Double augmentation: $d' = 2d$.

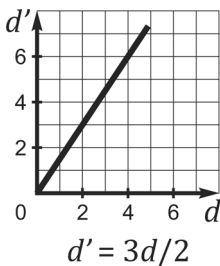


$d: 6$ 2 4 2 2 4
 P:

 ↓ ↓ ↓ ↓ ↓
 R:

 $d': 12$ 4 8 4 4 8

One-and-half augmentation ("the addition of a dot"): $d' = 3d/2$.

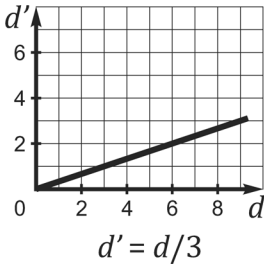


$d: 6$ 2 4 2 2 4
 P:

 ↓ ↓ ↓ ↓ ↓
 R:

 $d': 9$ 3 6 3 3 6

Triple diminution: $d' = d/3$.



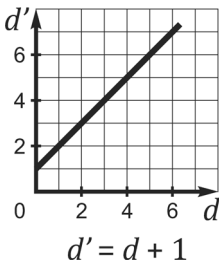
$d: 9 \quad 3 \quad 6 \quad 3 \quad 3 \quad 6$
 P:
 R:
 $d': 3 \quad 1 \quad 2 \quad 1 \quad 1 \quad 2$

Thus, the formula $d' = ad$ generalizes all *classical* transformations of durations: exact repetition, augmentation, and diminution. At the same time an a may be a quite “exotic” number, as in music by Conlon Nancarrow. Graph of any classical transformation is a straight line starting from the origin of the coordinate axes (0, 0) at an angle of φ . For exact repetition, $\varphi = 45^\circ$. For augmentation, $45^\circ < \varphi < 90^\circ$. For diminution, $0^\circ < \varphi < 45^\circ$.

2.2. Nonclassical Transformations

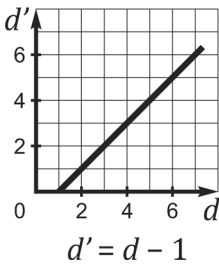
All other transformations of durations are *nonclassical*. A few examples.

(a) $d' = d + 1$. Addition of a duration. Here to each duration is added.



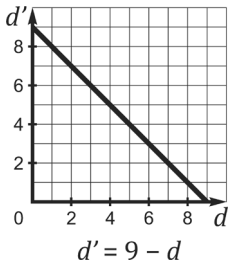
$d: 1 \quad 2 \quad 1 \quad 4 \quad 3 \quad 3 \quad 3 \quad 3 \quad 1$
 P:
 R:
 $d': 2 \quad 3 \quad 2 \quad 5 \quad 4 \quad 4 \quad 4 \quad 4 \quad 2$

(b) $d' = d - 1$. Subtraction of a duration. From each duration is subtracted.



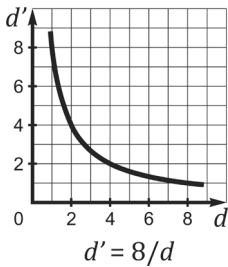
$d: 1 \quad 2 \quad 1 \quad 4 \quad 3 \quad 3 \quad 3 \quad 3 \quad 1$
 P:
 R:
 $d': 0 \quad 1 \quad 0 \quad 3 \quad 2 \quad 2 \quad 2 \quad 2 \quad 0$

(c) $d' = 9 - d$. *Converse relation of durations.* The longest possible duration is 9, it becomes 0 in a rистopa: $9 \rightarrow 0$; $8 \rightarrow 1$, $7 \rightarrow 2$ etc.; $0 \rightarrow 9$.



$d: 1 \quad 8 \quad 7 \quad 6 \quad 5 \quad 5 \quad 4 \quad 4$
 P:
 R:
 $d': 8 \quad 1 \quad 2 \quad 3 \quad 4 \quad 4 \quad 5 \quad 5$

(d) $d' = 8/d$. *Inverse proportional relation of durations.*

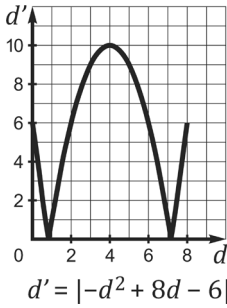


$d: 8 \quad 8 \quad 8 \quad 8 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 8 \quad 4 \quad 4 \quad 1 \quad 1 \quad 1 \quad 1$
 P:
 R:
 $d': 1 \quad 1 \quad 1 \quad 1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 1 \quad 2 \quad 2 \quad 8 \quad 8 \quad 8 \quad 8$

When working with such transformation, it is convenient to have a correspondence table as a visual aid. An upper row of a table contains durations of the proposta which are planned for use. A lower row comprises related durations of the rистopa.

	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\frac{3}{2}$	2	$\frac{8}{3}$	3	4	$\frac{16}{3}$	6	8	$\frac{32}{3}$	12	16
$d:$																
$d':$																

(e) $d' = |-d^2 + 8d - 6|$. Different durations of the proposta may be related to equal durations of the rистopa. For instance, 1 or $7 \rightarrow 1$; 0 or 2 or 6 or $8 \rightarrow 6$. This relation is not biunique. In other words, the inverse rule cannot unambiguously transform the rистopa back into the proposta. In the correspondence table the “inconvenient” values are rounded. The exact values are placed in parentheses.



$d: 1\ 1\ 1\ 1\ 6\ 7\ 1\ 1\ 7\ 1\ 7\ 1\ 7\ 1\ 7$

P:

R:

$d': 1\ 1\ 1\ 1\ 6\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$

	0	1/2	3/4	1	2/3	3/2	2	3/3	3	4	5	1 1/3	6	6 1/2	6 2/3	7	7 1/4	7 1/2	8
$d:$																			
$d':$																			
	6	2	1/2	1	3	4	6	8	9	10	9	8	6	4	3	1	1/2	2	6
		(2/4)	(3/16)		(2%)	(3 3/4)		(8%)				(8%)		(3 3/4)	(2%)		(1/16)	(2/4)	

(f) $d' = 2$. Equalized durations. All durations of the proposta turn into eighths. Backward transformation is not possible.

$d' = 2$

$d: 3\ 1\ 1\ 7\ 4\ 0\ 1/2\ 1/2\ 1/2\ 1/2\ 1/2\ 1/2\ 1/2\ 1/2\ 2/3\ 2/3\ 2/3$

P:

R:

$d': 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2$

3. TRANSFORMATIONS OF PITCHES

Transformations of pitches only, with other parameters remaining unaltered, can generally be defined as $(d, p', n) = f(d, p, n)$ or in short form $p' = f(d, p, n)$.

In this section we examine transformations where independent values are pitches only: $p' = f(p)$.

To assign numerical values to pitches we do the following:

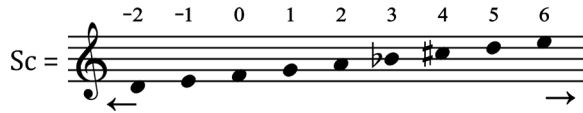
- (1) Define a scale S_c (some ascending row of pitches);
- (2) Take any pitch of this scale as a datum (0). The pitches to the left of this point have values $-1, -2, -3...$ in succession. The pitches to its right have values $1, 2, 3...$ in succession.

For brevity we write:

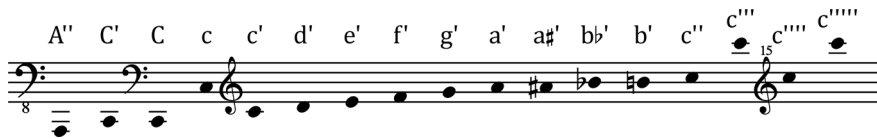
“ $S_c = \text{Chrom}, g' = 0$ ” for

$S_c =$

and “Sc = D minor Harmonic, f' = 0” for



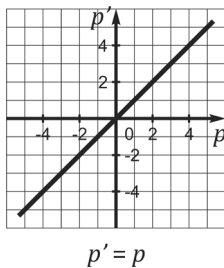
It is assumed that all pitches of a melody are members of the selected scale. Letter designations for pitches are:



A continuous line in the graphs is merely a convention. Only integers, for both p and p' , have practical significance.

3.1. Classical Transformations

Exact repetition. Pitches of the proposta and the rистпоста are the same. This is expressed by the formula $p' = p$.

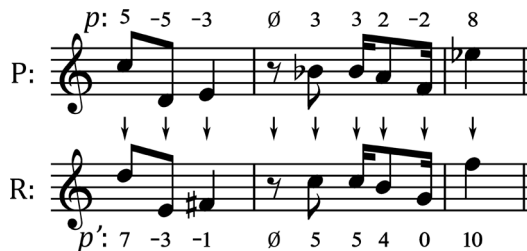
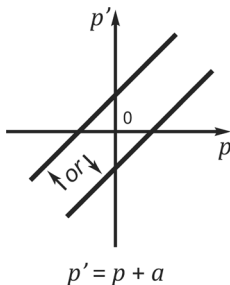


Sc = Chrom, $g' = 0$

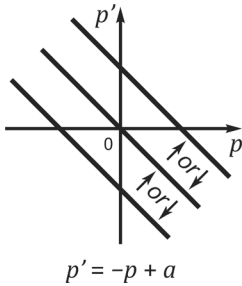


Transposed repetition is expressed by the formula $p' = p + a$ (a is an integer).

Sc = Chrom, $g' = 0$; $p' = p + 2$



Inversion is expressed by the formula $p' = -p + a$ (a is an integer).



$$Sc = \text{Chrom}, g' = 0; p' = -p$$

$p: 5 \quad -5 \quad -3 \quad \emptyset \quad 3 \quad 3 \quad 2 \quad -2 \quad 8$
 P:
 $p': -5 \quad 5 \quad 3 \quad \emptyset \quad -3 \quad -3 \quad 2 \quad -8$

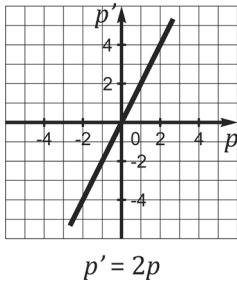
These are classical transformations of pitches. The general formula is $p' = \pm p + a$ (a is an integer). A graph of such transformations is a straight line inclined at the angle of 45° (for exact or transposed repetition) or of -45° (for inversion).

3.2. Nonclassical Transformations

All other transformations of pitches are nonclassical. Some examples.

(a) $p' = 2p$. *Melodic augmentation*. All intervals are increased by a factor of two.

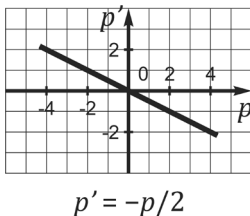
$$Sc = \text{Chrom}, g' = 0$$



$p: 5 \quad -5 \quad -3 \quad \emptyset \quad 3 \quad 3 \quad 2 \quad -2 \quad 8$
 P:
 $p': 10 \quad -10 \quad -6 \quad \emptyset \quad 6 \quad 6 \quad 4 \quad -4 \quad 16$

(b) $p' = -p/2$. *Melodic diminution and inversion*. All intervals are decreased by a factor of two and inverted. The proposta may contain even pitches only. In the case of 12-tone scale odd pitches turn into “quarter-tones”.

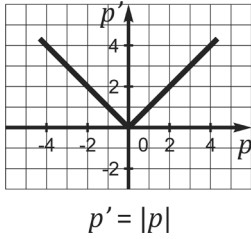
$$Sc = \text{Chrom}, a^\sharp = 0$$



$p: 2 \quad -8 \quad -6 \quad \emptyset \quad 0 \quad 0 \quad -2 \quad -6 \quad 4$
 P:
 $p': -1 \quad 4 \quad 3 \quad \emptyset \quad 0 \quad 0 \quad 1 \quad 3 \quad -2$

(c) $p' = |p|$. *Partially mirrored relation of pitches.* a' and higher pitches remain the same; pitches below a' become inverted about it. Backward transformation is not possible.

Sc = Chrom, $a' = 0$

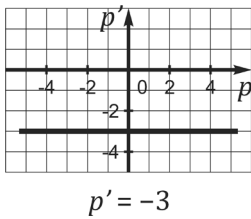


P: $p: 3 \quad -7 \quad -5 \quad \emptyset \quad 1 \quad 1 \quad 0 \quad -4 \quad 6$

R: $p': 3 \quad 7 \quad 5 \quad \emptyset \quad 1 \quad 1 \quad 0 \quad 4 \quad 6$

(d) $p' = -3$. *Equalized pitches.* All sounding pitches of the proposta turn into a' . Backward transformation is not possible.

Sc = Chrom, $c'' = 0$

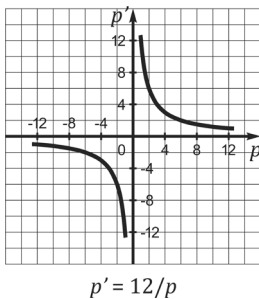


P: $p: 0 \quad -10 \quad -8 \quad \emptyset \quad -2 \quad -2 \quad -3 \quad -7 \quad 3$

R: $p': -3 \quad -3 \quad -3 \quad \emptyset \quad -3 \quad -3 \quad -3 \quad -3 \quad -3$

(e) $p' = 12/p$. *Inverse proportional relation of pitches.* The proposta may have only the following pitches: $-12, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 12$.

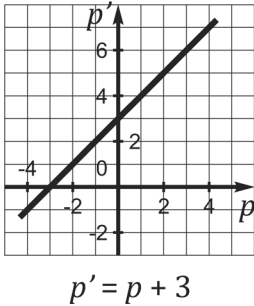
Sc = Chrom, $c' = 0$



P: $p: 1 \quad 3 \quad 4 \quad 3 \quad 6 \quad 4 \quad 3 \quad 1 \quad -1 \quad -2 \quad -4 \quad 12$

R: $p': 12 \quad 4 \quad 3 \quad 4 \quad 2 \quad 3 \quad 4 \quad 12 \quad -6 \quad -3 \quad 1 \quad -12$

(f) $p' = p + 3$. *Transposition*. Even though this is “just” a transposition, the transformation is quite noticeable since the pitches g' and c'' are considered as neighbouring degrees of the scale.



Sc =

P: $p: 4 \ 2 \ 2 \ 2 \ 3 \ 2 \ 4 \ 1 \ 3 \ 2 \ 3 \ 2 \ 3 \ 5$

R: $p': 7 \ 5 \ 5 \ 5 \ 6 \ 5 \ 7 \ 4 \ 6 \ 5 \ 6 \ 5 \ 6 \ 8$

4. TRANSFORMATIONS OF DURATIONS AND PITCHES IN COMBINATION

$$d' = d^2 - 3d + 3$$

$$p' = p^2 - 3p$$

Sc = D minor Harmonic, $f' = 0$

	1	2	3	4
d:				
d':				
	1	1	3	7

$p: -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ \emptyset$

$p': 10 \ 4 \ 0 \ -2 \ -2 \ 0 \ 4 \ 10 \ \emptyset$

$p: -2 \ 2 \ 1 \ 0 \ 0 \ 2 \ 5 \ 4 \ 5 \ \emptyset$

$d: 4 \ 2 \ 3 \ 1 \ 2 \ 2 \ 4 \ 2 \ 2 \ 2$

$d': 7 \ 1 \ 3 \ 1 \ 1 \ 1 \ 7 \ 1 \ 1 \ 1$

$p': 10 \ -2 \ -2 \ 0 \ 0 \ -2 \ 10 \ 4 \ 10 \ \emptyset$

5. TRANSFORMATIONS OF INTERVALS

Often it is more convenient and musically more natural to manipulate melodic intervals, not absolute values of pitches. In this case a note is considered as (d, i, n) , i is an interval from the previous sounding note.

To assign numerical values to intervals, it is sufficient to define a scale. Any pitch of this scale might be taken as a datum, but in fact there is no need to do so at this point. It is enough to state that: (1) the difference between neighbouring degrees of the scale is equal to one; (2) an ascending interval is positive, a descending interval is negative, a unison is zero.

All sounding pitches of a melody, except the first one, may be defined by means of intervals. *The first sounding pitch* (sp_1) is to be defined by its absolute value.

Examples.

Sc = Chrom, $sp_1 = c''$

$p: c''$
 $i: \emptyset -10 2 \quad \emptyset 6 0 -1 -4 10$

Sc = F major, $sp_1 = c''$

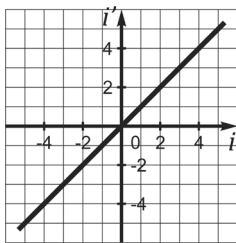
$p: c''$
 $i: \emptyset -6 1 \quad \emptyset 4 \emptyset 0 -1 -2 5$

$i = \emptyset$ is a *vacant interval*, i. e. a rest.

In this section we examine transformations where independent and dependent values are intervals only: $i' = f(i)$. The first sounding pitch of the rispostia we denote as sp'_1

5.1. Classical Transformations

Exact or transposed repetition. Intervals of the proposta and the rispostia are the same. This is expressed by the formula $i' = i$.



$$i' = i$$

Sc = Chrom, $sp'_1 = c''$

$i: -10 2 \quad \emptyset 6 0 -1 -4 10$

P:

R:

$i': -10 2 \quad \emptyset 6 0 -1 -4 10$

Sc = Chrom, $sp'_1 = d''$

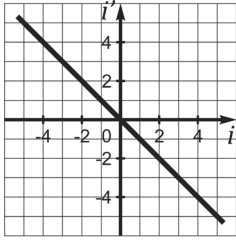
$i: -10 2 \quad \emptyset 6 0 -1 -4 10$

P:

R:

$i': -10 2 \quad \emptyset 6 0 -1 -4 10$

Inversion is expressed by the formula $i' = -i$.



$$i' = -i$$

$$Sc = \text{Chrom}, sp'_1 = d'$$

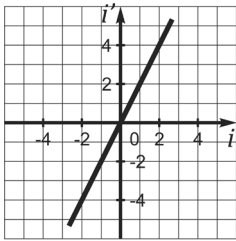
$i:$ -10 2 \emptyset 6 0 -1 -4 10
 P:
 R:
 $i':$ 10 -2 \emptyset -6 0 1 4 -10

We can see that for the same transformations the formulas $p' = f(p)$ and $i' = f(i)$ are often different.

5.2. Nonclassical Transformations

Some examples.

(a) $i' = 2i$. *Melodic augmentation*. All intervals are increased by two.

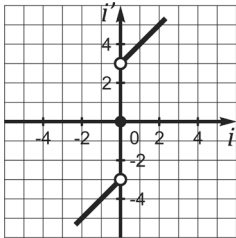


$$i' = 2i$$

$$Sc = \text{Chrom}, sp'_1 = f''$$

$i:$ -10 2 \emptyset 6 0 -1 -4 10
 P:
 R:
 $i':$ -20 4 \emptyset 12 0 -2 -8 20

(b) $i' = i + \text{sgn}(i) \times 3$. *Addition of an interval*. All intervals but unison are increased by the absolute value of 3. (Function $\text{sgn}(i)$ is defined as follows: $\text{sgn}(i) = -1$ if $i < 0$; $\text{sgn}(i) = 1$ if $i > 0$; $\text{sgn}(i) = 0$ if $i = 0$.)

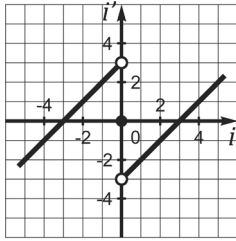


$$i' = i + \text{sgn}(i) \times 3$$

$$Sc = \text{Chrom}, sp'_1 = c''$$

$i:$ -10 2 \emptyset 6 -1 -2 0 -3 10
 P:
 R:
 $i':$ -13 5 \emptyset 9 -4 -5 0 -6 13

(c) $i' = i - \text{sgn}(i) \times 3$. *Subtraction of an interval*. All intervals greater than or equal to minor third are decreased by the absolute value of 3. The direction of the intervals less than minor third, but unison, becomes opposite.



$$i' = i - \text{sgn}(i) \times 3$$

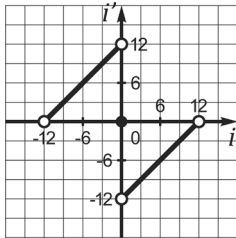
Sc = Chrom, $sp'_1 = c''$

$i:$ -10 2 \emptyset 6 -1 -2 0 -3 10

P:

R: $i':$ -7 -1 \emptyset 3 2 1 0 0 7

(d) $i' = i - \text{sgn}(i) \times 12$. *Inversion of the intervals within an octave*.



$$i' = i - \text{sgn}(i) \times 12$$

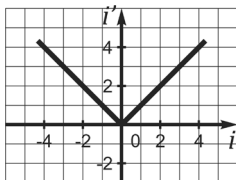
Sc = Chrom, $sp'_1 = c''$

$i:$ -10 2 \emptyset 6 0 -1 -4 10

P:

R: $i':$ 2 -10 \emptyset -6 0 11 8 -2

(e) $i' = |i|$. *Partially mirrored relation of intervals*. All intervals except the unison become ascending.



$$i' = |i|$$

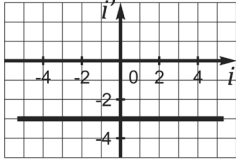
Sc = Chrom, $sp'_1 = c$

$i:$ -10 2 \emptyset 6 0 -1 -4 10

P:

R: $i':$ 10 2 \emptyset 6 0 1 4 10

(f) $i' = a$. Equalized intervals.



$i' = -3$

$Sc = Chrom, sp'_i = a''$

$i: -10 \quad 2 \quad \emptyset \quad 6 \quad 0 \quad -1 \quad -4 \quad 10$

P:

R:

$i': -3 \quad -3 \quad \emptyset \quad -3 \quad -3 \quad -3 \quad -3 \quad -3$

$Sc = Chrom, sp'_i = a'$

$i: -10 \quad 2 \quad \emptyset \quad 6 \quad 0 \quad -1 \quad -4 \quad 10$

P:

R:

$i': 0 \quad 0 \quad \emptyset \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

6. PERMUTATIONS OF NOTES

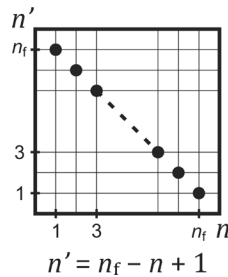
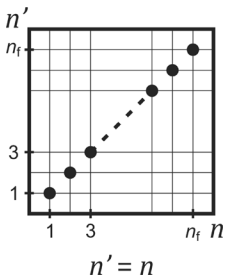
Transformations of sequence numbers only (i. e. permutations of notes), with other parameters remaining unaltered, can generally be defined as $(d, p, n') = f(d, p, n)$ or in short form as $n' = f(d, p, n)$.

In this section we examine transformations where independent values are numbers only: $n' = f(n)$.

The notes in a melody are numbered sequentially starting with 1. The rest is considered as a note. If there are several successive rests in a music score, they are considered (and numbered) as a single rest. The last note of a melody is denoted by n_f .

6.1. Classical Transformations

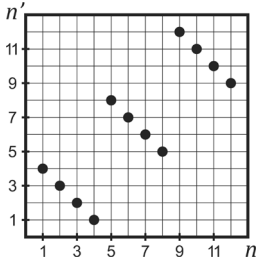
There are only two classical transformations: *direct motion* ($n' = n$) and *retrograde motion* ($n' = n_f - n + 1$).



6.2. Nonclassical Transformations

Some examples.

(a)



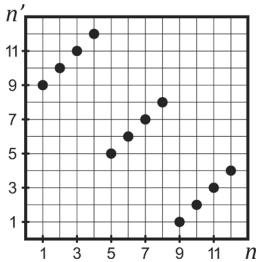
Example (a) musical notation showing P (Primo) and R (Ritornello) staves with mapping arrows and indices n and n'.

P: $n: 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

R: $n': 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

$n: 4\ 3\ 2\ 1\ 8\ 7\ 6\ 5\ 12\ 11\ 10\ 9$

(b)



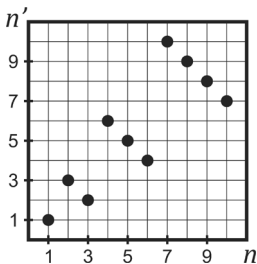
Example (b) musical notation showing P (Primo) and R (Ritornello) staves with mapping arrows and indices n and n'.

P: $n: 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

R: $n': 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

$n: 9\ 10\ 11\ 12\ 5\ 6\ 7\ 8\ 1\ 2\ 3\ 4$

(c)



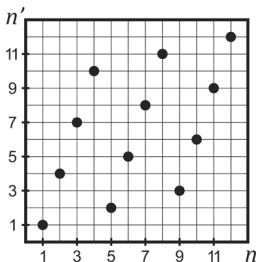
Example (c) musical notation showing P (Primo) and R (Ritornello) staves with mapping arrows and indices n and n'.

P: $n: 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10$

R: $n': 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10$

$n: 1\ 3\ 2\ 6\ 5\ 4\ 10\ 9\ 8\ 7$

(d)



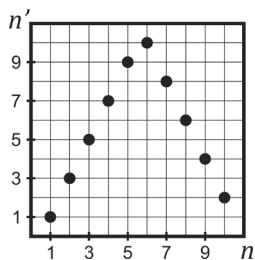
Example (d) musical notation showing P (Primo) and R (Ritornello) staves with mapping arrows and indices n and n'.

P: $n: 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

R: $n': 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

$n: 1\ 5\ 9\ 2\ 6\ 10\ 3\ 7\ 11\ 4\ 8\ 12$

(e)



n: 1 2 3 4 5 6 7 8 9 10

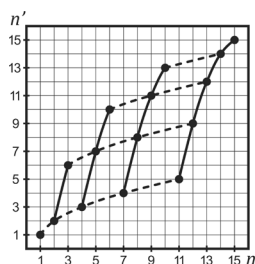
P:

R:

n': 1 2 3 4 5 6 7 8 9 10

n: 1 10 2 9 3 8 4 7 5 6

(f)



n: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

P:

R:

n': 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

n: 1 2 4 7 11 3 5 8 12 6 9 13 10 14 15

7. TRANSFORMATIONS OF DURATIONS, PITCHES, AND NUMBERS IN COMBINATION

$$d' = d^2 - 3d + 3$$

$$p' = p^2 - 3p$$

$$n' = 7 - n \text{ if } n \leq 6$$

$$n' = 17 - n \text{ if } n \geq 7$$

Sc = D minor Harmonic, f' = 0

n:	1	2	3	4	5	6	7	8	9	10
p:-	2	1	0	0	2	5		4	5	∅
d:	4	2	3	1	2	2	4	2	2	2

P:

R:

d':	1	1	1	3	1	7		1	1	1	7
p':	-2	0	0	-2	-2	10		∅	10	4	10
n':	1	2	3	4	5	6		7	8	9	10
n:	6	5	4	3	2	1		10	9	8	7

8. PERMUTATIONS OF DURATIONS AND PITCHES SEPARATELY

By ${}_a n$ and ${}_a n'$ we denote the sequence numbers of original and derived durations; by ${}_p n$ and ${}_p n'$ we denote the sequence numbers of original and derived pitches.

A simple example: the order of durations remains the same, the order of pitches becomes reversed.

$${}_a n' = n$$

$${}_p n' = n_t - n + 1$$

9. TRANSFORMATIONS BETWEEN VALUES OF DIFFERENT DIMENSIONS

(a) Durations turn into pitches and vice versa.

$$d' = p$$

$$p' = d$$

$$Sc = C \text{ major}, c' = 1$$

(b) Intervals become durations.

$$d' = d \text{ for the first note}$$

$$d' = |i| \text{ for other notes}$$

$$Sc = C \text{ major}$$

(c) Numbers turn into pitches and vice versa.

$$d' = d$$

$$p' = n$$

$$n' = p$$

Sc = C major, c' = 1

n:	1	2	3	4	5
p:	5	1	3	4	2

P:

R:

p':	2	5	3	4	1
n':	1	2	3	4	5
n:	2	5	3	4	1

10. TRANSFORMATIONS OF SEVERAL INDEPENDENT VARIABLES

(a) Duration of the risposta depends on two different durations.

$$d'_n = d_n \text{ if } n = 1$$

$$d'_n = d_n + d_{(n-1)} \text{ if } n > 1$$

d:	4	4	1	2	1	2	2
----	---	---	---	---	---	---	---

P:

R:

d':	4	8	5	3	3	3	4
-----	---	---	---	---	---	---	---

(b) Duration of the risposta depends on a proposta's duration and pitch: $d' = f(d, p)$.

$$d' = d + p$$

Sc = C major, c' = 1

p:	5	2	1	2	4	2	3	2	2	1	5
d:	4	1	3	2	6	2	3	1	1	1	5

P:

R:

d':	9	3	4	4	10	4	6	3	3	2	10
p':	5	2	1	2	4	2	3	2	2	1	5

(c) Independent values may belong to different propostas. The following example demonstrates an “exchange of durations”.

¹ d:	4	4	6	2	6	4	1	21
² d:	4	4	4	4	4	4	4	20

¹P, ²P:

¹R, ²R:

¹ d':	4	4	4	4	4	4	4	20
² d':	4	4	6	2	6	4	1	21

Etc.

11. EXAMPLES TAKEN FROM MUSIC PRACTICE

The following cases may be understood not only as rhythmical but also as completely strict pitch imitations:

J. S. Bach. From "Fugue in D major" for organ, BWV 532

The transformation $d' = a$ can be found in the music by Béla Bartók.

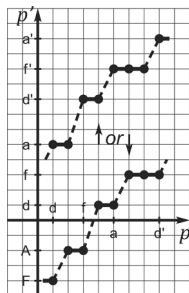
From "Music for Strings, Percussion and Celesta", IV, 1936

The transformation $p' = a$ can be found in the music by Alfred Schnittke.

From "The Fourth Violin Concerto", II, 1984

The transformations $d' = d + a$ (addition of a duration) and $d' = 2(d + a)$ (addition of a duration and double augmentation) are used in the "Canon in Three Voices" by Vassily Lobanov from "Seven Slow Pieces" for piano, Op. 34, 1980 [6].

"Trivium" for organ, 1976, by Arvo Pärt contains the following transformations:



Similar transformations may be found in other works by the composer.

Permutations of notes, durations, and pitches are often associated with twelve-tone technique [4, 148–150] and with the music by Olivier Messiaen [1, 175–178; 5, 134–142].

Repetition with omission of rests and of all small notes is found in the mass “*Allez regretz*”, *Agnus II* by Josquin des Prés (the authorship is doubtful) [3, 151]. This device is considered as a form of canonic technique in the music of the 15th–16th centuries.

Many examples of melodies composed on the basis of *algorithmic procedures* which in fact are strict transformations as well, may be found, in particular, in music and books by Tom Johnson (for instance [2]).

“*Four Canons*” were written by the author of these lines in 1981. In these canons the following nonclassical transformations are used: $d' = 8/d$; $d' = 2$; $d' = d + 3$; $d' = -d^2 + 8d - 6$; $i' = i + \text{sgn}(i) \times 7$; ${}^2p' = {}^1p' + |i| - 4$. I have never seen any of them before. (See https://nv.mosconsv.ru/sites/default/files/2023-07/Sergei%20Zagny_Four%20Canons.pdf and https://youtu.be/w_gYzm_CqnQ.)

12. CONCLUSION

The proposta and the risposta reflect each other. They are kin. In simple cases such as repetition, their kindred relationship is obvious. For augmentation, inversion, and reverse motion this is less clear. The more complex the transformation, the less evident is the kinship of the proposta and the risposta. Observing music from the outside, we are not always able to realize whether such a relationship actually exists. Even similar things are often similar not because of kinship but by coincidence.

D. Scarlatti. From “Sonata in D major”, K. 96. – P. Tchaikovsky. From “The First Piano Concerto”, I, Op. 23.



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